

# **DYNAMIC SIMULATION OF A BOILING WATER REACTOR NUCLEAR POWER PLANT FOR LARGE TRANSIENTS**

**A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of  
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**by  
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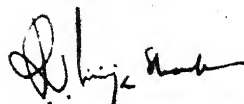
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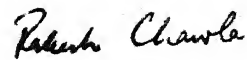
# CERTIFICATE

Certified that the present work, entitled "Dynamic Simulation of a BWR Power Plant for Large Transients", has been carried out by Shri P.K. Malhotra under our supervision and has not been submitted elsewhere for the award of a degree.



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## NOMENCLATURE

$A_C$	Cross-sectional area of the core met by water flow ( $\text{ft}^2$ )
$A_{DR}$	Cross-sectional area of the drum in the plane of free water surface ( $\text{ft}^2$ )
$A_H$	Total fuel-clad surface area, exposed to coolant flow ( $\text{ft}^2$ )
$A_{K2}$	Constant relating the flow and pressure drop in a turbine of many stages
$A_1, A_2$	Cross-sectional area for main and secondary steam flow at the throttle valves ( $\text{in}^2$ )
$A(x_C),$ $A(x_R)$	Two phase flow frictional factors in the core and risers respectively
$C$	Delayed neutron precursors (W)
$C_p$	Specific heat of water (W/lbm - $^{\circ}\text{F}$ )
$C_1, C_2$	Coefficients used in the doppler reactivity relations
$C_1^*, C_2^*$	Coefficients used in main and secondary steam flow expressionsthrough throttle valve
$E_0, E_1,$ $E_2$	Coefficients relating efficiency of the recirculating pump to flow
$f_C$	Leakage flow factor
$g_c$	Gravitational constant (lbm ft./lbf-sec $^2$ )
$H_F$	Heat transfer coefficient for fuel (W/ft $^2$ - $^{\circ}\text{F}$ )

$H_{FW}$	Heat transfer coefficient of feed water heaters (WS/lb)
$H_R$	Heat transfer coefficient of reheater (WS/lb °F)
$h_f$	Enthalpy of saturated water at pressure P (W - sec/lbm)
$h_{fg}$	Enthalpy of saturated steam at pressure P (W - sec./lbm)
$h_C, h_R,$ $h_S, h_2$	Enthalpy of main steam at nozzle chest, reheater, throttle valve, moisture separator and isentropic end points from pressure $P_C$ and $P_R$ respectively (WS/lb.)
$h_f, h'_{fw},$ $h_{fw}, h_o$	Enthalpy of saturated water in the reheater feedwater leaving heater 1, heater 2 and water in the condenser respectively (WS/lb)
$J$	Joules constant relating mechanical energy and heat energy (WS/ft lb)
$K_I$	Coefficient used in momentum equation
$k_1, k_{2C},$ $k_{3C}, k_{2D}$	Friction factors
$K_{BHP},$ $K_{BLP}$	Coefficients indicating bled steam from HP and LP turbine
$K_1, K_2$	Calender's empirical state equation constants $\text{lbf ft}^3 / (\text{m}^2 \text{ WS}), \text{lbf ft}^3/\text{m}^2$
$L_C$	Length of the core (ft)
$L_D$	Length of the downcomer (ft)

$L_{DR}$	Length of the steam drum (ft)
$L_F$	Length of the fuel rods (ft)
$L_R$	Length of the riser (ft)
$L_{WL}$	Water level length in the steam drum (ft)
$l$	Prompt neutron life time (sec.)
$m_f'$	Mass of water in two phase region of core, riser and steam drum in lbm
$m_g$	Mass of steam in the reactor consisting of core, riser and drum in lbm.
$N$	Neutron power (W)
$n$	Number of fuel rods used in the core
$P, P_C, P_R$	Pressure in reactor, nozzle chest, and reheater (lbs/sq. in)
$Q$	Heat transfer flow rate in the reactor core (Watts)
$Q_{H1}, Q_{H2}$	Heat transferred in heater No. 1 and 2 respectively (WS)
$Q_R$	Heat transferred in reheater (W)
$Q_{SUB}$	Sub-cooled heat absorbed by water in raising the temperature from core inlet to saturation (W)
$r$	Radius of casing (ft)
$R$	Universal gas constant (lbf ft <sup>3</sup> / m <sup>2</sup> lbm °F)
$S_C, S_R$	Slip in the core and riser respectively
$T_{BS}$	Boiling length transit time
$T_D$	Water temperature entering the lower plenum chamber (°F)

$T_F, T_{F,o}$	Average fuel temperature at any time and under steady state
$T_{FW}$	Feed water temperature $^{\circ}F$
$T_I$	Core inlet temperature $^{\circ}F$
$T_{HP}, T_{LP}$	Torque of high pressure and low pressure turbine respectively (lbf ft)
$T_{H1}, T_{H2},$ $T_{R1}, T_{R2},$ $T_{W2}, T_{W3}$	Time constants associated with heater No. 1, heater No. 2, reheaters for flow rate and heat transfer rate, and with HP and LP turbine respectively (sec.)
$T_M$	Temperature of water after mixing in the lower plenum chamber ( $^{\circ}F$ )
TORQ	Net torque at the turbine shaft (lbf ft)
$T_S$	Saturation temperature of water at a pressure P ( $^{\circ}F$ )
t	Time (sec.)
$V_C, V_R$	Total effective volume of the nozzle chest and reheater respectively (ft <sup>3</sup> )
$V_f, V_g$	Specific volume of saturated water and steam respectively around a pressure of 200 psi (Cft/lbm, Cft/lbm)
$V_{in}$	Core inlet velocity (ft/sec.)
$V_{fex}$	Core exit velocity of water (ft/sec.)
$V_{fR}$	Velocity of water in the riser (ft/sec.)
$V_{gex}$	Core exit velocity of steam (ft/sec.)



$V_{gR}$	Velocity of steam in the riser (ft/sec)
$V_{LP}$	Volume of the lower plenum chamber (ft <sup>3</sup> )
$W_1, W_2,$ $W_3, W_2',$ "	Steam flow rate at throttle valve, nozzle chest, LP turbine, before and after the reheater respectively (lbm/sec.)
$W_{BHP},$ $W_{BLP}$	Bled steam from high pressure and low pressure turbine unit respectively (lbm/sec.)
$W_D, w_D$	Volume and weight flow rate of recirculating water (ft <sup>3</sup> /sec., lbm/sec. respectively)
$w_{FW}$	Weight flow rate of feed water (lbm/sec.)
$W_{HP}', W_{MS}'$	Weight flow rate of water in heater 2 and moisture separator (lbm/sec.)
$W_L$	A constant which depends on ratio of turbine flow required to produce a net torque to rated flow of steam
$W_{PR}, W_{PR}'$	Secondary steam flow rate entering and leaving respectively (lbm/sec.)
$W_S, w_S$	Volume and weight flow rate of steam (ft <sup>3</sup> /sec., lbm/sec.)
$x_C$	Average quality of steam in the core
$x_{ex}$	Core exit quality of steam
$x_R$	Quality of steam in the riser
$\alpha_C$	Average void fraction in the core
$\alpha_R$	Void fraction in the riser
$\alpha_{ex}$	Core exit void fraction

$\alpha_S$	Core void fraction corrected for void
$\beta$	Yield of delayed neutron group
$\Delta k$	Total excess reactivity
$\Delta k_{DOP}$	Doppler reactivity feedback ( $\$/^{\circ}F$ )
$\Delta k_{ex}$	Externally introduced reactivity through control rods ( $\$$ )
$\Delta k$	Void reactivity coefficient
$\Delta P_{ACC}$	The required pressure difference to accelerate the two phase mixture (psi)
$\Delta P_F$	Frictional losses in primary loop (psi)
$\Delta P_H$	Buoyancy force due to the density difference around the loop (psi)
$\Delta P_P$	Pressure developed by the recirculating pump in psi
$\lambda$	Decay constant ( $sec^{-1}$ )
$\rho_g$	Specific weight of steam ( $lbm/ft^3$ )
$\rho_t$	Specific weight of water ( $lbm/ft^3$ )
$\tau$	Time delay from the drum to the inlet to lower plenum chamber (sec.)
$\tau_{F1}, \tau_{F2}$	Time constants used in the fuel dynamics
$\eta_{HP}^* \eta_{LP}^*$	Isentropic efficiency of the HP and LP turbine respectively
$\eta_{CFHP}$	Efficiency correction factors to take into account the reduction of rotational losses etc. at reduced loads for HP and LP turbine respectively.
$\eta_{CFLP}$	

$\rho_C, \rho_R, \rho_2$  Density of steam in the nozzle chest,  
reheater, and at the turbine outlet  
(lbs/cft)

$\Omega$  Speed of the turbine equal to  $120 \pi$  (rad/sec.)

## TYPICAL VALUES USED FOR THE REACTOR PARAMETERS

$A_C$	$= 10.2 \text{ ft}^2$	$T_{FW}$	$= 441048$
$A_R$	$= 4.56 \text{ ft}^2$	$T_{R1}$	$= 3.0 \text{ Sec.}^{-1}$
$A_{DR}$	$= 240 \text{ ft}^2$	$T_{R2}$	$= 4.0 \text{ Sec.}^{-1}$
$A_{K2}$	$= 3.1111944$	$T_{H1}$	$= 100 \text{ Sec.}^{-1}$
$\beta$	$= 0.0068$	$T_{H2}$	$= 40.0 \text{ Sec.}^{-1}$
$\bar{C}_1$	$= 0.112 \times 10^{-2}$	$T_{H2}'$	$= 60.0 \text{ Sec.}^{-1}$
$\bar{C}_2$	$= 2.3883995 \times 10^{-7}$	$T_R$	$= 515^\circ\text{F}$
$C_P$	$= 1340 \text{ W/lbm} - ^\circ\text{F}$	$V_{LP}$	$= 300.0 \text{ ft.}^3$
$f_C$	$= 0.75$	$V_C$	$= 85.0 \text{ ft.}^3$
$G_C$	$= 32.2 \text{ ft/sec}^2$	$V_R$	$= 9500 \text{ ft.}^3$
$H_{FW}$	$= 3.8519228 \times 10^3 \text{ W}_S/\text{lb}$	$\eta^*$	$= 0.86$
$H_Q$	$= 130 \text{ W}_S/\text{lb}^\circ\text{F}$	$F1$	$= 6.6 \text{ sec.}$
$k_C$	$= 0.6608$	$F2$	$= 6.6 \text{ sec.}$
$k_D$	$= 9.93$	$T_m$	$= 1988.7 \text{ lbf} - \text{ft}^*$
$K_1$	$= 179.2/644 \times 1.055 \times 10^3)$	$n_p$	$= 1660/60 \text{ rps}^*$
$K_2$	$= 149670/144$	$J_P$	$= 11000 \text{ lb} - \text{ft}^{2*}$
$k_R$	$= 0.0298$		
$k_n$	$= 52.0$		
$L_C$	$= 6.2 \text{ ft}$		
$l$	$= 5 \times 10^{-4} \text{ Sec.}$		
$L_F$	$= 5.833333 \text{ ft.}$		
$n$	$= 8008$		
$L_R$	$= 63.0 \text{ ft}$		
$L_{DR}$	$= 5.1 \text{ ft}$		

\* Data referred to BWR/6 type

## SYNOPSIS

Mathematical non-linear models for the boiling water reactor and the turbine sub-assemblies, studied earlier, have been modified and assembled to represent the Big Rock Point nuclear power plant in order to obtain better plant response, in the present work. Modifications introduced are (i) the non-linear representation of the throttle valve (ii) the incorporation of the recirculating pump dynamics in the reactor recirculation loop (iii) representation of the voids by first order sweep model. To gain confidence in this modified plant model, tests like pump trip, control rod oscillation are simulated on the digital computer and are compared with the plant test results. Many other tests, whose plant test results were not available, are also simulated for better understanding of the plant response. It would be observed that the present model behaved favourably while comparing with the available test results. The thesis also reviews the work done in the field of modelling and optimal control techniques by the researchers and also presents recommendations for further work.

## CHAPTER 1

### INTRODUCTION

#### 1.1 IMPACT OF NUCLEAR POWER GENERATION

The biggest problem of the world today is either to search new sources of energy or improve upon the already existing available sources of energy to cope up with the fast growing demand for energy. The sources of energy like coal, petroleum, geothermal, gas etc. may not be able to meet this growing demand mainly because of the fast deteriorating world reserve for these fuels. This leads one to think of the nuclear energy as an alternative to these energy sources. Today, both the developed and developing countries are striving in a big way to master this source of energy. For instance it is expected that in U.S. in 1985 the contribution by this source would be 37% of the total power against 0.5 percent in 1970. In India also two nuclear power stations are already in operation at Tarapur and Rajasthan and two more at Kalpakam and Narora would come up very soon.

#### 1.2 TYPES OF NUCLEAR POWER PLANTS

The major constituents of a nuclear power reactor are the fuel and the moderator situated in the core where controlled fission takes place. The fission in the

fuel gives 2.5 fast neutrons at energies greater than 1 MeV for every neutron absorbed in the  $U^{235}$  nucleus (average). The energy of fast neutron is degraded by elastic scattering in the moderator from 1 MeV to 0.025 eV approximately. The neutrons which are in thermal equilibrium with the assembly are called thermal neutrons (approx. 0.025 eV) and are ready for further fissioning to maintain the chain cycle. This process is achieved in different ways which entirely depends on the type of fuel and moderator employed. It is known that natural uranium has smaller fission probabilities using light water as moderator. So enriched fuel has to be used to build critical assemblies while building nuclear reactor with light water as the moderator. These reactors are called as light water reactors which can be subgrouped as (i) boiling water reactor where coolant is at low pressure and boiling is permitted and (ii) pressurised water reactors where coolant is at high pressure where bulk boiling is not permitted. Natural uranium can be used as fuel with heavy water as the moderator which absorbs very little amount of neutrons. This is the CANDU system.

Gas cooled graphite moderated reactors are popular in United Kingdom and advance gas cooled heavy water reactors concept is being developed at U.K. to achieve higher efficiency upto 40% or so. These reactors

are basically thermal reactors since neutrons at thermal energies are used for fissioning and to maintain chain reaction.

Another class of reactors is the fast reactor where the neutrons at higher energy (1 MeV) are used in maintaining the chain reaction. This precludes the use of moderators. The liquid coolant like sodium, potassium etc. are used to remove the energy from the reactors mainly due to their higher thermal conductivity property. In fast reactors source of the neutrons can be captured in fertile materials to produce the fissile material which after processing can be used as fuel again. These are called as breeder reactors and have been developed by France and Russia.

### 1.3 OBJECTIVE OF THE PRESENT STUDY

The objective of this thesis is to study and combine the models of the Big Rock Point boiling water reactor and turbine sub-systems, separately studied earlier. On this the complete plant model tests like R C Pump failure, control rod oscillation etc. are simulated to compare the response with the test results in an effort to validate the model. Since comparisons between the two are poor, modifications are introduced on this combined model to get a better match. Besides, other tests are also studied



to give an insight into the plant behaviour. The content of each Chapter is briefly discussed below.

The Chapter 2 discusses the details of Big Rock Point boiling water reactor. The experimental results used in this thesis for comparison pertains to this reactor. The literature in the area of dynamics and control of nuclear reactors is surveyed in great details in this chapter.

Chapter 3 is devoted to the development of a mathematical model based on physical laws and empirical relations. Previous work on this subject is briefly discussed and modifications of this model are presented in detail.

The dynamic simulation of the model given in Chapter 3 for various disturbances forms the basis of the Chapter 4. Comparison of the plant test responses with the model responses for some tests such as pump trip, control rod etc. is also given. Finally the chapter is concluded with the recommendations for further work.

## CHAPTER 2

### DESCRIPTION, CONDUCTED TESTS AND OTHER SIMULATION STUDIES FOR BRP POWER PLANT

#### 2.1 GENERAL FEATURES SITE AND LOCATION

The Big Rock Point nuclear power plant site is located in Charlevoix country, between the towns of Charlevoix and Petoskey, on the northern shore of Michigan's lower peninsula. The site property includes 600 acres of gently sloping wooded and cleared land on the shore of Lake Michigan at the western extremity of the southern shore of Little Traverse Bay. The site is 228 miles NNW of Detroit and 262 miles NNE of Chicago.

#### 2.2 PLANT FEATURES

The Big Rock Point nuclear power plant consists of a direct cycle, forced circulation boiling water reactor, a power extraction system and associated service facilities. The principal structures include:

- i) a 130 ft. diameter spherical containment vessel,
- ii) a turbine building,
- iii) a structure housing water intake facilities,
- iv) a 240 ft. stack waste storage vaults.

The containment vessel houses the reactor, recirculation piping, pumps, steam drum, fuel pool, and equipment for

removal of shut down heat. The turbine-generator and other conventional plant equipments are housed in a separate building. The containment is a spherical steel vessel of 130 ft. dia, its prime purpose being to prevent any harmful spread of radioactive material to the environment in the event of an accident in the reactor system. It also serves as a weather-proof housing.

### 2.3 PLANT DESCRIPTION

For detailed description and design criteria one should refer to references [1 - 3]. The major components (Fig. 2.1) are briefly described below.

#### REACTOR VESSEL

The vessel is cylindrically shaped with full hemispherical top and bottom as shown in the Figure 2.2. It is 30 ft. long, 106 inch in diameter with wall thickness 5.25 inch and cladding thickness 5/32 inch. It is designed for 1700 psia pressure, 650 °F temperature, whereas normal operating conditions could be 1500 psia, 593 °F.

#### REACTOR VESSEL PENETRATIONS:

<u>Penetrations</u>	<u>Number</u>	<u>Diameter</u> (inches)	<u>Location</u>
Coolant water inlets	2	20	Bottom
Steam-water mixture outlets	6	14	Shell
Outlet to shutdown heat exchanger	1	6	Shell

Access ports in top head	3	10	Head
Control-rod drive penetrations	32	4	Bottom
Liquid poison inlet	1	3	Bottom
Emergency core spray inlet	1	3	Shell
Vessel vents	2	3	Head
In-core flux monitors penetrations	8	2	Bottom
Instrument nozzles	4	3	Shell
Seal leak monitor	1	1/2	Head flange

#### FUEL AND CORE DESCRIPTION

Each fuel bundle contains 144 fuel rods in square array, 132 being standard rods and 12 being special rods used to minimize the local hot spot factors. The fuel channels surrounding each fuel bundle provide guide surfaces for the control rods and allow orificing of the flow between channels. The channels are supported by support tubes. There are 32 control rods cruciform in nature ( $11 \frac{1}{2}$  inch wide x  $5/16$  inch thick) consisting of 116 poison tubes containing  $B_4C$  powder. When control rod movement fails, scram is initiated by pilot valves to make the reactor sub-critical till it cools down, the liquid poison (850 gallons) being located above the steam drum and consisting of sodium pentaborate solution ( $Na_2 B_{10} O_{16} \cdot 9 H_2O$ ) obtained by dissolving boric acid and borax at  $212^\circ F$ .

## BIOLOGICAL SHIELDING

To protect plant personnel from radiation hazards due to neutron and  $\gamma$  - radiation, a biological shield is provided based on the international dose acceptance level of 5 rem per year. It is made up of heavy concrete and is provided with necessary cooling.

## STEAM DRUM

The steam drum is situated high up in the enclosure. Its main function is to separate steam from the steam-water mixture from the reactor and provide exit steam of 99.9% quality. With its storage capacity of 500 cubic feet, surges of water level and pressure between the reactor vessel and the drum are easily accommodated. It also assures NPSH (net positive suction head) for the recirculating pumps. Water level in the drum is 65 feet above the pump suction level and provides adequate natural circulation driving heads, when the pumps are free to rotate but not being driven by the electric motor coupled to it. It will be seen (Sec.4.1.1) that the reactor under such condition returns to 50% of normal power. The drum has an axial length of 40 ft. with ID of 78 inches.

## RECIRCULATING PUMPS

There are two vertically mounted recirculating pumps operating in parallel, each capable of returning 17000 gpm of water to the core. The pumps are single-stage, centrifugal, double-suction with an overhung impeller. The pump driver is a vertical, drip-proof induction motor of conventional design, rated for 400 HP at 1600 RPM, 2300 volts.

## PRIMARY LOOP PIPING AND VALVES

The primary loop piping interconnects all the major components of the nuclear steam supply system. Six 14 inch risers carry the steam-water mixture from the reactor vessel to the steam drum. After the steam is separated, the recirculating water flows out of the drum down four 17 in. downcomers. The downcomers, in two groups of two each, join together into two 24 in. pump suction headers. The two pump discharges are each 20 in. and return to the bottom of the reactor vessel. A system of vibration dampers are fastened to the piping to reduce possible vibration stresses. An open bypass between the suctions of the two pumps will keep water flowing in the four downcomers even if one pump fails essentially, overriding the possibility of tilting of the drum due to unequal expansion in the two sets of downcomer piping. Each pump has 24 in. gate valve at suction and 20 in. gate valve at

discharge end to keep out of service if leakage becomes excessive. In addition a 20 in. butterfly valve at the discharge end regulates flow during research and development programmes. When the valves are fully closed, scram is initiated. Six safety valves are located on the main steam drum meant for operation beyond 1870 psia pressure in the drum.

#### TURBINE GENERATOR EQUIPMENT

The turbine is a 3600 rpm, tandem compound, double flow, condensing unit, directly connected to a hydrogen cooled generator which in turn is connected through a reduction gear to an air-cooled exciter. Three points of extraction for feed water heating are provided. The turbine is rated at 54,500 KW at 1000 psig. 0 degree final superheat and  $3 \frac{1}{2}$  in. of mercury absolute exhaust pressure, with 3% make-up allowance and three feedwater heaters in service. The turbine is capable of operating continuously at 1450 psig, 0 degree final superheat and 2 inches Hg back pressure with a maximum expected output of 75,000 KW. The 13,800 volts, wye-connected generator is rated at 70,588 KVA, 0.85 power factor, 0.8 short circuit ratio at 30 psig hydrogen cooling pressure.

Besides conventional design criteria, all modifications necessary due to use of saturated steam from

the BWR are incorporated in the turbine design. Particular attention is paid to eliminate pockets or crevices in which radioactive material may lodge. Each turbine stage is drained either internally or externally. The turbine is provided with moisture removal buckets ahead of each extraction point. In addition, two external moisture separators are provided in the cross-over between the high pressure and low pressure sections.

#### MAIN TURBINE CONDENSER

The main turbine condenser performs the following functions:

- a) ~~condenses~~ the steam exhausted from the turbine to obtain the desired heat utilization and vacuum,
- b) de-aerates the condensate and water from heater drains and other returns,
- c) serves as a heat-sink for excess reactor steam dumped through the turbine by-pass valve, and
- d) detains condensate in the hot well to permit decay of short lived radioactivity.

The ~~condenser~~ is made-up of steel and is a horizontal, single pass, divided water-box de-aerating type unit of conventional construction. It provides 27,500 square feet of effective condensing surface area. A 6000 gallon oversized, baffled, storage-type hot well is provided to allow decay of short-lived radioactivity.



## CONDENSATE AND FEED-WATER SYSTEM AUXILIARIES

### a. EXTRACTION DRAINS AND VENTS

Extraction steam for feedwater heating is taken from three points off the turbine to the respective heaters. The two higher pressure extraction lines are provided with automatic bleeder trip valves to protect the turbine from flooding in the event of a heater tube break or overspeed from steam flashing out of the heater after a turbine trip.

Water collected from the turbine moisture removal stages is piped to the drain cooling section of either the high, intermediate, or low pressure heaters. Heater drains are cascaded to the condenser where they are de-aerated and merge to the condensate flow.

### b. CONDENSATE PUMPS

Two half-capacity, vertical, multistage centrifugal pump, pumps the condensate from the hot well through the condensate system to the suction of the reactor feed pumps.

### c. FEEDWATER PUMPS

Two feedwater pumps take suction directly from the condensate system, discharge feedwater through the high pressure heater and through a common header to the reactor steam drum. These are horizontal, multistage, centrifugal pumps.

#### d. FEEDWATER HEATERS

Three feedwater heaters are located in the condensate circuit. The flow pressure and intermediate pressure heaters are of the horizontal mounted, U - tube type with removal tube bundles, integral drain coolers, and bolted head covers.

#### TURBINE AND BYPASS CONTROL SYSTEM

The turbine obeys two modes of control:

- a. Initial pressure regulation (IPR, Base-load operation)
- b. Speed control (House-unit operation).

Normally, the Turbo-Generator set is base loaded and to avoid swings with accompanying transients being felt by the reactor, the steam main line pressure is maintained constant by IPR. This essentially, controls valve position without regard to the generator or transmission line system loads. But in the event of excessive speed build-up, IPR is overridden. This excessive build-up normally happens when the 138 KV transmission line trips (due to some transmission faults) and load is rejected on the generator actuating turbine speed-up.

#### AUXILIARY COOLING WATER SYSTEMS

The service water system is an open system in which strained water is supplied from pumps in the intake structure and returned to the lake along with the discharge

from the circulating water system, essentially, to cool the following equipment:

- Generator hydrogen coolers,
- Turbine tube oil coolers,
- Feed pump bearings and oil coolers,
- Air compressor, after-coolers and jackets,
- Miscellaneous sample coolers,
- Airconditioning system,
- Reactor enclosure air coolers,
- Space heating and cooling equipment,
- Reactor Cooling water heat exchangers.

#### REACTOR COOLING WATER SYSTEM

This is a closed intermediate cooling loop utilizing demineralised water to cool the following equipments:

- Reactor shield cooling panels,
- Reactor clean-up Non-regenerative heat exchanger,
- Miscellaneous sample coolers,
- Reactor recirculating pump coolers.

#### 2.4 DESCRIPTION OF TESTS CONDUCTED ON THE PLANT

These tests are extensively documented in references [4, 5] and are summarized below.

## CONTROL ROD OSCILLATION TESTS

The aim was to measure the closed loop, reactivity -to-neutron-flux transfer function of the primary hydraulic loop for various conditions of flow, power and pressure. One of the standard control rods was made to oscillate in the frequency range 0.2 - 5.0 cps, while keeping reactor steam pressure controller, steam - drum level controller and the movement of control rods (other than the oscillating rod) on manual operation to avoid unwanted variation of parameters due to controller cycling.

## NOISE TESTS

The aim was to collect data of the noise in steady state flux and flow signals after performing rod oscillation test. These tests were of long duration (45 minutes) following immediately after CRO test. Feedwater level controller and reactor steam pressure controller were put on manual operation to eliminate noise due to controller cycling effects.

## PRESSURE OSCILLATION TESTS

These were performed to determine the pressure-to-neutron-power transfer function of the reactor primary hydraulic loop. The main steam valve bypassing the turbine was oscillated over the frequency range 0.02 - 3.0 cps.

## PRESSURE TRANSIENT TESTS

These tests were performed before control rod oscillation tests. The drum level controller was put on manual. Reactor pressure was controlled by first opening the turbine bypass valve by 3% and then changing this set point rapidly. The pressure, neutron flux, steam flow and feedwater flow were continuously monitored during the test.

## PUMP TRIP TEST

The tests discussed so far involved small order perturbation, while the pump trip test and the others that are discussed below involved large-order changes.

For various operating values of pressure, temperature and neutron power, the power to both recirculation pumps was tripped and signals were continuously recorded for reactor pressure, neutron power, average heat flux, core inlet velocity and core inlet temperature. There were several variations of the pump trip test, e.g., tripping of a single pump, restart of the pumps after the trip etc.

## FLOW CONTROL TEST

With the reactor operating at rated flow and at a certain power level, the recirculation flow was decreased in steps to 90, 80, 70, 60 and 50 percent of

the rated flow without the control rod movements. The data so collected helped in thorough understanding of core performance.

#### LOAD FOLLOWING TEST

With the plant operating at specified operating point, the turbine was put on governor control and feed-water on automatic operation. Electrical load was rejected from the turbine at rates of 1, 2, 3, 5 MWt per minute. For each rate, pressure regulation was obtained by manual throttling of the butterfly valves in the recirculation loop. During this test, control rods were held at fixed positions.

#### NATURAL CIRCULATION TEST

This test was conducted for the 84 bundle core configuration only at 1350 psia. The plant was initially operating at 75 MWe at rated two-loop forced circulation. The control rods were inserted in a certain specified pattern. After the reactor steadied out, the pumps were tripped till power and flow coasted down to their natural circulation levels. Thereupon, power level increases were affected in accordance with a specified programme of control rod movements. For each new control-rod pattern, data was obtained to calculate the reactor power from the heat balance equations. Prior to each power

increases, a comparison was made between the actual natural circulation flow and power conditions and the licensed heat transfer limit. When the comparison indicated that it was permissible, further power increases were made until the heat transfer limit was nearly reached.

## 2.5 SAFETY STUDIES

### DYNAMIC STABILITY OF THE REACTOR

Some of the low pressure ( $\approx 500$  psi) experimental BWR's exhibit power oscillations due essentially to mutual feedback between hydrodynamic flow oscillations and the effect of voids on reactor power. The design criteria for BRP which ensure stable operation include high system pressure, forced circulation, a conservative void reactivity coefficient, a long fuel heat-transfer time constant and the use of a pressure regulator to control system pressure. The analytical studies reported in GEAP-3795 [6] based directly on the physical concepts of momentum interchange, energy conservation and mass continuity, also do rule out the possibility of any such instabilities.

Usual unsafe operating conditions for the plant would be short reactor period, high neutron flux, high reactor pressure, low water level in the primary steam drum, simultaneous closure of turbine stop valves and primary by-pass valves, closure of backup primary steam

sphere isolations valves, high enclosure pressure, low condenser vacuum, low water level in the reactor vessel, simultaneous closure of the recirculation water line valves, and high level in the scram dump tank. At the onset of such potentially unsafe conditions, the reactor is automatically scrammed and cooling initiated through the emergency condenser.

The experimental tests discussed in Sec. 2.4 serve to varify the inherent stability of the plant. However, one should consider the safety requirements in the event of equipment malfunctioning, operator error or some combination of these.

#### SAFETY DESIGN AGAINST EQUIPMENT MALFUNCTION

The design requirements for safety necessitate the following:

- i) use of equipment with the highest practical reliability against failure,
- ii) fail-safe design, i.e. any failure will result in action toward safe direction,
- iii) safety through redundant devices (mostly triplicate system employing 2/3 redundancy, is in vogue).

The possible important equipment failures could be:



## CONTROL ROD MALFUNCTION

The following may help in its initiation:

(i) failure of normal drive power, (ii) failure of emergency drive power, (iii) separation of rod from drive.

In all such cases, reactor is shut down through scrams.

## LOSS OF ELECTRICAL LOAD

A sudden loss of electrical load will cause a partial closure of the turbine control valve. The bypass steam valve will open automatically to hold reactor system pressure constant by providing a path to the condenser for the turbine-rejected steam. Thus, there will be no undesirable flux transient from the load change. The control rods would be manually repositioned to hold reduced power. In the event of bypass valve failure, or its slow operation, the reactor will be scrammed through the high neutron flux monitor or through a high pressure signal bringing up emergency condenser action, in case the flux monitor fails. In the event of complete load rejection the reactor is scrammed if both the turbine and bypass valve remains closed.

## LOSS OF CONDENSER VACUUM

If the condenser vacuum is lost, the main reactor heat sink is lost. If no action were taken by an automatic protection system the condenser pressure would increase to a point where the condenser rupture diaphragm

would fail to ease off this built pressure. This possibility is eliminated by sending a scram signal to the reactor through a vacuum sensing device. Also the turbine is tripped. Otherwise, if the device is inoperative, closure of the bypass and stop valves is demanded and this introduces scram directly or indirectly through high pressure. If the cause were loss of coolant flow due to power failure, the reactor receives a shutdown signal through an overall energised safety system.

#### FUEL CLADDING FAILURE

An off-gas hold-up system provides protection when fuel cladding fails. The system is designed for manual closure of the gas holder isolation valve at a noble gas concentration that could be reasonably expected to deliver the maximum permissible off site dose if the operation were to continue unabated for a year. Automatic shutdown will occur at a greater gas release rate in time to keep personnel exposures within acceptable limits.

#### LOSS OF FEEDWATER HEATERS

Sudden loss of all the feedwater heaters would cause an immediate but smooth rise in flux. Within a few minutes the flux would be high enough to initiate scram, and no overshoot or damage to fuel would result.

## LOSS OF FEEDWATER

Loss of feedwater will result in gradual lowering of the water level in the steam drum and, if continued, will automatically initiate reactor shut down. Main condenser cools the reactor adequately.

## SAFETY DESIGN AGAINST OPERATOR ERROR

The probability of accidents being initiated by operator mistakes is very less, considering the level of training administered to reactor operators. Nevertheless, safety is ensured by providing detailed operating and maintenance procedures, and by special design measures for control of certain operations. For example, the reactor is never started up unless all safety systems have been properly actuated. Further, only one control rod can be withdrawn at a time and that too at a limited rate. In PSAR [2] a number of hypothetical situations are discussed which pessimistically combine operator error with equipment failures to result in

- a) start up accident,
- b) cold water accident,
- c) fuel loading accident,
- d) closure of steam line back-up isolation valve,
- e) failure to replenish cooling water in emergency condenser.

It is shown how such accidents are rendered impossible by the various safety systems in BRP.

#### SIMULATED ACCIDENTS

WASH - 1270 [7] and WASH - 1400 [8] describe a variety of simulated maximum credible accidents for nuclear power plants, resulting from

- a) primary system rupture,
- b) sphere pressure reduction,
- c) leakage from enclosure,
- d) large LOCA and small LOCA.

It is shown in these reports that such occurrences have extremely low probabilities. References [9, 10] on the maintenance and operational record of BRP are proof of the excellent safe performance of the power plant to date.

#### 2.6 REVIEW OF EARLIER LWR SIMULATION AND CONTROL STUDIES

A non-linear model of the reactor and turbine for load following studies of BRP has been presented in IBM research report [11]. GIRIJASHANKAR [12, 13, 14] considered the simplified 8 - order linear version of the nuclear reactor to simulate the small order disturbances separately and then by regular approach studied feedback control laws that drove the system to steady state.

GIRIJASHANKAR [15] also studied an 11 - order linear model of the nuclear turbine for small order disturbances. LINFORD [16] made a GEC BWR model for transient studies. GLAASSEN and ECKERT [17], in the suggested direction of WASH - 1270 [7] simulated tests without scram to see the plant response of a BWR more rigorously. Earlier BECKER [18, 19] studied burnout conditions in channels both experimentally and analytically. BECKER et al. [20] also pointed out conditions for hydrodynamic instability. LEONARD, SUN, MUNTHE ANDERSON and DIX [21] developed a GEC model for analytical LOCA studies for reactor safety evaluation. MEHTA [22] studied non-linear simulation of an organic-cooled Canadian CANDU reactor. CHOU [23] considered a detailed model of Pickering G.S. 'A' for load following studies. KERLIN [24] simulated H.B. ROBINSON BWR nuclear reactor for reactivity and steam valve perturbations. ATRAY and SHAH [25] modeled a complete prototype power plant for analytical studies.

Such studies of modelling and simulation necessarily validate the model before conducting optimal control studies. FROGNER [26] has highlighted the importance of optimal control studies and in his Ph.D. thesis [27] has given a useful review. He himself applied feed-forward technique on a 13 order linear model derived from 25 order non-linear differential equations using

Kalman filter technique for non estimable state variables to implement Direct Digital Control on BWR. The algorithm was tested on estimation of void reactivity coefficient under different conditions by RAO [28]. FROGNER [29] has surveyed optimal control methods and discussed [30] identification techniques in the presence of plant noise. GRUMBACH [31] stressed the simultaneous need of software and hardware control structures less sensitive to component failure in control and instrumental failure. He built up the conceptual design for such techniques. Modern concepts are more or less based on Kalman Filter techniques and most of the work simplified to date is nucleated around these. Thus, GODBOLE [32] studied the application of Kalman filter technique in nuclear reactors to obtain the non-measurable variables. He later [33] applied Kalman filter technique in the presence of white process contaminated with noise.

VENEROUS and BULLOCK [34] used Kalman filter technique to estimate time dependent reactivity. But this technique increased the computational burden, and SAMANT and SORENSON [35] devised a low-order filter to save computer time. Little attended observers were studied LUENBERGER [36, 37]. GODBOLE [38] designed a new procedure which (a) took care of linear feedback laws without sacrificing observer poles, (b) had a true deadbeat

observer which tracks the actual states in observability under time steps.

Presently, research interests are to apply process computers for the optimal on-line control of nuclear power plants. LUNDE [39, 40] has discussed the development and use of such hardware techniques. BLOMSNES [41, 42] considered the power behind linear state variable feedback control while considering spatial core distributions. GRUMBACH [43] proposed the computer usage at Halden for controlling the spatial distribution of neutron flux. GRUMBACH [44] analysed plant disturbances through process computers. GRUMBACH and BLOMSNES [45] presented practical applications at Halden. BLOMSNES and NETLAND [46] proposed the use of a simulator and trained operators for computer based reactor control. JOSEFSSON [47] in a cooperative effort between Sweden and Norway, showed the successful use of advanced control methods at Studsvik through the simulator STUDS-1 where load-follow control studies were conducted on the Nordic electric power grid. BJORLE et al. [48, 49], BLOMSNES et al. [50] GRUMBACH [51] have reported the recent application of optimal control laws to HBWR at the OECD Halden Project, highlighting the advantage of DDC control over conventional methods.

## OBJECTIVE OF THE PRESENT WORK

Most of the work reported till date ignores the turbine dynamics or represents it in a simplified form, as MANKIN and SHINOHARA [52]. This is good enough when boiler transients are initiated by weak disturbances and persist only for small duration. But for large order disturbances lasting for minutes, the turbine dynamics cannot be ignored or represented by its associated time constants by simple equations. The present effort is concentrated to develop the analytical non-linear model, introducing suitable modifications, for the BWR power plant suitable for large order disturbances.

This kind of models could be useful in

- a) building simulator employing real-time process computer to train the operator to have good feel for both normal and abnormal operation and the plant behaviour,
- b) to conduct optimal control studies,
- c) to check the plant behaviour under anticipated transients without scram.

The present study discusses the nonlinear model of the turbine and the reactor subsystems separately. Then on the combined model which represents the boiler and turbine, the response for the large order transients are studied. The BRP boiling water reactor plant data will be used to validate the model.



## CONCLUSIONS

The important components of BRP boiling water reactor and the tests conducted on it were discussed. Some of these will be simulated and discussed in Chapter 4. A useful literature survey giving information on modelling and control technique was listed. Finally the object of this work was outlined.

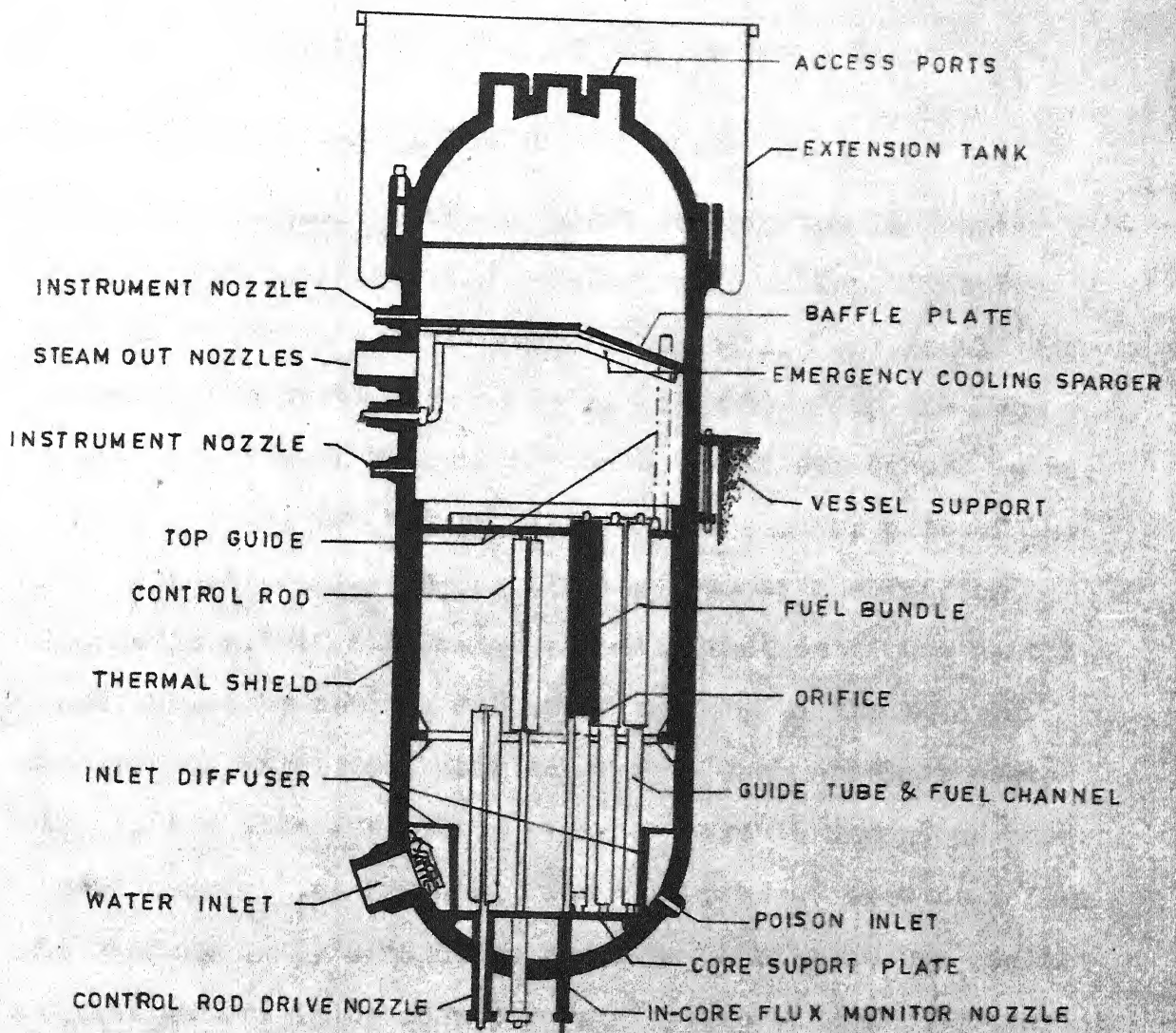


FIG.2.2 BIG ROCK POINT REACTOR VESSEL INTERNALS

## CHAPTER 3

### NON - LINEAR MODEL STUDIES FOR BRP POWER PLANT

#### 3.1 GENERAL CHARACTERISTICS

We dealt with the plant description in Chapter 2 for the BRP boiling water reactor. The salient features of its operation are reviewed. For details one should refer to the texts such as Refs. [53 - 55]. In the BWR core, the fission process of the  $U^{235}$  nucleus occurs in the fuel releasing 200 MeV per fission. The major part of the heat generation is via the slowing down of the fission fragments within the fuel. A small fraction of the heat is contributed by neutron and gamma heating of the coolant/moderator, structural materials etc. The neutrinos escape the system with a fraction of irrecoverable amount of the total energy i.e. 200 MeV. The transport of heat from fuel to casing is by conduction and from casing to the coolant (light water) is by convection. This heating of the coolant converts the single phase coolant into a two phase mixture in the core. Steam in this two phase mixture is separated by steam separators in the drum when the two phase mixture advances into the drum. From here the steam is allowed to flow into the turbine as shown in Fig. 3.1. Once the steam is separated in the drum the remaining saturated water in the recirculation flow mixes with the feedwater

(subcooled) in the drum and then flows down the downcomer as subcooled water to enter the inlet of the lower plenum chamber and then the core through the recirculating pumps. 20% of the recirculating water at inlet cools the control rods while the rest 80% cools the fuel elements.

Steam from the drum enters the steam chest of the turbine through the throttle valve and expands in the HP unit, a part of this steam is bled out at exit. At this point the moisture separators trap the moisture to increase the quality and returning the collected moisture to heater No. 2 while the main steam enters reheater. The reheater superheats the steam which then expands through the IP & LP units. A part of this steam is bled out and the rest is passed to the condenser. The condenser also receives the main steam through by-pass valve in emergency conditions and in conditions where tests like valve rod oscillations are conducted. The condenser operates at uniform pressure (assumption in our study). The water coming out of condenser is heated in reheaters No. 1 and 2 by bled steam out of IP & LP, and HP unit. This heated feedwater returns to the drum where it takes 6 seconds to mix-up with the saturated water of the recirculation flow.

### 3.2 BASIC MODEL RELATIONSHIP

These physical processes are translated to mathematical language (Sec. 3.3) through four fundamental laws viz.:

#### a. CONSERVATION OF ENERGY

Energy is neither created nor destroyed, if spent in one form it reappears in other forms. This essentially implies that the time rate of change of energy in a control volume is given by the balance between its generation rate within the control volume, the entrance rate from the surroundings and the exit rate to the surroundings.

#### b. CONSERVATION OF MOMENTUM

The net force of an inertial layer is equal to its rate of change of momentum.

#### c. CONSERVATION OF MASS

For each component the continuity equation is preserved, i.e. the rate of mass increase in a control volume is equal to mass flow rate at entrance less that at exit.

#### d. EQUATION OF STATE

For any thermodynamic cycle which consists of thermodynamic process of only  $PdV$  type of work mode

there are only two independent variables such as pressure and enthalpy. Any other variable can be represented in term of these two variables, Reynolds [58]. This is the State Postulate Law.

### 3.3 PREVIOUS MODELS

#### 3.3.1 THE REACTOR

GIRIJASHANKAR [12, 13] essentially considered the IBM non-linear model [11], and linearised it for the control studies of the reactor. The basic development was in accordance with Sec. 3.2, with the following assumptions:

- (i) The model does not calculate minimum critical heat flux ratio (MCHFR) or the maximum fuel temperature in coolant channel, local velocity changes and local void changes etc. Fast transients were not included.
- (ii) The entire neutron population responds as one energy group. The spatial variations of the neutron flux are ignored, and a single delayed-neutron group being considered in representing the core kinetics. Only the reactivity feedback due to changes in fuel temperature and void changes are considered and no control rod sequential pattern insertion is studied.
- (iii) The core average void fraction is a constant fraction of the core-exit void fraction.

- (iv) The temperature distribution of the fuel is independent of axial and angular space variables. Fuel density and specific heat are constant across the fuel element. Storage of heat in fuel-clad gap or heat generation in the clad is negligible.
- (v) In the hydrodynamic study, the variables like core steam quality and void fraction vary instantaneously with the changes in coolant flow, coolant inlet temperature and heat flow to the coolant. Pressure in the steam drum is spatially invariant. The steam and water phases are always in thermodynamic equilibrium.
- (vi) In the derivation of the momentum equation, compressibility effects in the loop are ignored. Slip, the ratio of average fluid velocity to the average steam velocity at core exit is assumed independent of void fraction and mass flow rate.
- (vii) The recirculation flow liquid is subcooled, incompressible and one-dimensional flow in the loop is assumed, i.e. the velocity heads in the bulk water, downcomer and lower plenum are in the direction of flow. The core leakage flow remains at the core inlet enthalpy until mixing occurs at the core outlet.

- (viii) The flashing process occurs instantaneously after pressure changes in the system when the two phases are in thermal equilibrium.
- (ix) A transport delay of 9 seconds between the drum and the inlet plenum chamber is included disregarding delay due to mixing in the steam drum.

With these assumption, the model so developed is represented as follows:

#### POWER DYNAMICS

$$\frac{dN}{dt} = \frac{\Delta k - \beta}{\lambda} N + \lambda C \quad (3.1)$$

$$\frac{dC}{dt} = \frac{\beta}{\lambda} N - \lambda C \quad (3.2)$$

where

$$\Delta k = \Delta k_{ex} - \Delta k_{DOP} - \Delta k_{\alpha} \quad (3.3)$$

$$\Delta k_{DOP} = \left( C_1 + \frac{C_2 Q}{A_H} \right) (T_F - T_{F,o}) \quad (3.4)$$

$$\Delta k_{\alpha} = \frac{23}{2} (\alpha_{c,o}^2 - \alpha_c^2) \quad (3.5)$$

$$A_H = 2\pi r n L_F \quad (3.6)$$

#### FUEL DYNAMICS

$$\frac{dT_F}{dt} = \frac{N}{\tau_{F1} A_H H_F} - \frac{1}{\tau_{F1}} (T_F - T_S) \quad (3.7)$$

$$\frac{dQ}{dt} = \frac{1}{\tau_{F2}} (N - Q) \quad (3.8)$$



## CORE HYDRO-DYNAMICS

$$\frac{d V_{in}}{dt} = \frac{1}{K_I \rho_f} \times [\Delta P_P + \Delta P_H - \Delta P_F - \Delta P_{ACC}] \quad (3.9)$$

$$\frac{d T_I}{dt} = \frac{W_D}{V_{LP}} (T_D - T_I) \quad (3.10)$$

where

$$\Delta P_P = POWP (E_0 - E_1 W_D - E_2 W_D^2) \quad (3.11)$$

$$\Delta P_H = (\rho_f - \rho_g) (L_C \alpha_C + L_R \alpha_R) \quad (3.12)$$

$$\begin{aligned} \Delta P_F = & (k_1 + k_{2C} L_C + k_{3C} L_C A(x_C) + k_{3R} \\ & + L_R A(x_R) + k_{2D} L_D) \frac{\rho_f V_{in}^2}{g_c} \end{aligned} \quad (3.13)$$

$$P_{ACC} = \left[ \frac{\{\rho_f (1 - \alpha_{ex}) + \rho_g \alpha_{ex} s_C^2\} \rho_f}{\rho_f (1 - \alpha_{ex}) + \rho_g \alpha_{ex} s_C^2} - 1 \right] \frac{\rho_f V_{in}^2}{g_c} \quad (3.14)$$

$$K_I = \left[ L_C + \frac{A_C}{A_R} L_R + \frac{A_C}{A_D} L_D \right] \frac{1}{g_c} \quad (3.15)$$

$$A(x_C) = 1 + 2400 \left( \frac{x_C}{14.7 P} \right)^{0.97} \quad (3.16)$$

$$A(x_R) = 1 + 2400 \left( \frac{x_R}{14.7 P} \right)^{0.97} \quad (3.17)$$

$$x_{ex} = \frac{Q - W_D \rho_f C_P (T_S - T_I)}{W_D \rho_f h_{fg} f_C} \quad (3.18)$$

$$x_C = 0.5 x_{ex} \quad (3.19)$$

$$x_R = f_C x_{ex} \quad (3.20)$$

$$\alpha_{ex} = \frac{\rho_f x_{ex}}{(1 - x_{ex}) \rho_g s_C + \rho_f x_{ex}} \quad (3.21)$$

$$\alpha_R = \frac{\rho_f x_R}{(1 - x_R) \rho_g s_R + \rho_f x_R} \quad (3.22)$$

$$\bar{\alpha}_C = 0.55 \alpha_{ex} \quad (3.23)$$

$$s_C = \frac{V_{gex}}{V_{fex}} \quad (3.24)$$

$$s_R = \frac{V_{gR}}{V_{fR}} \quad (3.25)$$

$$T_D(t) = T_M(t - \tau) \quad (3.26)$$

$$T_M - T_S = \frac{T_S - T_{FW}}{W_D \rho_f} W_{FW} \quad (3.27)$$

#### PRESSURE DYNAMICS

$$\frac{dP}{dt} = \frac{W_{FW} - W_S + \frac{\gamma}{h_{fg}} (Q - Q_{SUB})}{\eta + \frac{\gamma m_{f1} c_P}{h_{fg}} \frac{dT_S}{dP}} \quad (3.28)$$

where

$$Q_{SUB} = W_D \rho_f c_P (T_S - T_I) \quad (3.29)$$

$$\gamma = \frac{1}{\rho_g} - \frac{1}{\rho_f} \quad (3.30)$$

$$\eta = -m_g \left( \frac{d}{dP} \left( \frac{1}{\rho_g} \right) \right) \quad (3.31)$$

$$m_{f1} = \rho_f [L_{WL} A_{DR} + L_R A_R (1 - \alpha_R) + L_C A_C (1 - \bar{\alpha}_C)] \quad (3.32)$$

$$m_g = \rho_g [L_C A_C \bar{\alpha}_C + L_R A_R \alpha_R + (L_{DR} - L_{WL}) A_{DR} + L_{WL} A_R \alpha_R] \quad (3.33)$$

### FEEDWATER DYNAMICS

$$\frac{d w_{FW}}{dt} = k_{FW1} (w_{FW} - w_S) + k_{FW2} (L_{WL} - L_{WLO}) \quad (3.34)$$

where

$$L_{WL} = \left\{ \int_0^t (w_{FW} - w_S) dt + L_C A_C (\rho_f (\bar{\alpha}_C - \bar{\alpha}_{C,0}) + \rho_{g,0} \bar{\alpha}_{C,0} - \rho_g \bar{\alpha}_C) + L_R A_R [\rho_f (\alpha_R - \alpha_{R,0}) + \rho_{g,0} \alpha_{R,0} - \rho_g \alpha_R] + A_{DR} (L_{DR} (\rho_{g,0} - \rho_g) + L_{WLO} (\rho_f - \rho_{g,0})) \right\} / (A_{DR} (\rho_f - \rho_g)) \quad (3.35)$$

$$w_S = C_1 A_1 P \quad (3.36)$$

$$w_D = \frac{A_C V_{in}}{f_C} \quad (3.37)$$

$$w_{FW} = \frac{w_{FW}}{\rho_f} \quad (3.38)$$

$$w_S = w_S \rho_g \quad (3.39)$$

$$T_S = 544.6 + 0.125 (P - 1000) ^\circ F \quad (3.40)$$

$$\rho_f = 46.2 \text{ lbm/ft}^3 \quad (3.41)$$

$$\rho_g = 2.25 \times 10^{-3} P \text{ lbm/ft}^3 \quad (3.42)$$

$$h_{fg} = 6.85 \times 10^5 - 2.08 (P - 1000) \times 10^2 \text{ W Sec/lbm} \quad (3.43)$$

$$h_f = 5.72 \times 10^5 + 1.81 (P - 1000) \times 10^2 \text{ W Sec/lbm} \quad (3.44)$$

Equations 3.40 to 3.44 are empirical relations around 1000 psia deduced from Mollier charts and steam tables.

Equations 3.16 and 3.17 are due to Becker's experimental correlation [56] where he observed that 'A' is independent of mass flow rate. This, however is not strictly true, but for the present study we will use his formula for the two-phase friction multiplier.

### 3.3.2 TURBINE MODEL

GIRIJASHANKAR [15] compared the validity of non-linear and linear turbine models which were developed based on IBM report [11].

The development assumed the following:

- (i) steam inertia and kinetic energy changes are ignored,
- (ii) a perfect-mixing, adiabatic process prevails in the steam chest,

- (iii) the throttling is a pure isenthalpic process allowing no change of enthalpy across the valve. For a given valve stem position, the flow through the valve is proportional to the upstream pressure of the valve,
- (iv) bleed flow to the heaters for regeneration purposes is at reheater pressure and so is a fraction of instantaneous flow through the turbine. Neither representation of thermodynamic path nor calculations for individual stages are attempted,
- (v) the overall efficiency of the HP, LP and IP turbine is modified to yield a lumped efficiency. Correction factors are used to take into account efficiency changes due to stage leaks, root and trip losses, rotational losses and reheat effects,
- (vi) the dynamics associated with the intermediate and low pressure turbine are secondary to the dominant dynamics of the reheater. Thus IP & LP are represented in a lumped single unit,
- (vii) the condenser pressure is assumed to be invariant,
- (viii) full flow is used to produce net work through the HP turbine while one-half of the bleed flow produces torque in the IP and LP turbines. This is because bleeding is done at the end of HP turbine while in the IP & LP turbines, it is carried out at various points.

With these assumptions the following equations result:

#### NOZZLE BOWL DYNAMICS

$$\frac{d h_C}{dt} = \left[ \frac{(w_1 h_s - w_2 h_C)}{\rho_C v_C} + \frac{J P_C}{\rho_C^2} \frac{d \rho_C}{dt} \right] * \frac{1}{1 - k_1 J} \quad (3.45)$$

$$\frac{d \rho_C}{dt} = \frac{w_1 - w_2}{v_C} \quad (3.46)$$

$$w_1 = C_1^* A_1^* P \quad (3.47)$$

$$w_2 = A_{k2} (P_C \rho_C - P_R \rho_2)^{1/2} \quad (3.48)$$

$$x = \frac{h_2 - h_f}{h_{fg}} \quad (3.49)$$

$$P_C = \rho_C (k_1 h_C - k_2) \quad (3.50)$$

$$\rho_2 = \frac{1}{x v_g + (1 - x) v_f} \quad (3.51)$$

#### HP TURBINE DYNAMICS

$$TW_2 * \frac{dW_2''}{dt} = (W_2 - W_{BHP}) - W_2'' \quad (3.52)$$

$$W_{BHP} = K_{BHP} W_2 \quad (3.53)$$

#### MOISTURE TRAP

$$W_{MS} = (W_2 - W_{BHP}) - W_2' \quad (3.54)$$

$$W_2' = \frac{h_2 - h_f}{h_{fg}} W_2'' \quad (3.55)$$

## REHEATER

(a) MAIN STEAM

$$\frac{d \rho_R}{dt} = \frac{W_2' - W_3}{V_R} \quad (3.56)$$

$$\frac{d h_R}{dt} = \left\{ \frac{Q_R + W_2' h_g - W_3 h_R}{\rho_R V_R} + \frac{J \rho_R (W_2' - W_3)}{\rho_R^2 V_R} \right\} * * \frac{1}{1 - k_1 J} \quad (3.57)$$

$$P_R = \rho_R (k_1 h_R - k_2) \quad (3.58)$$

$$P_2 = \rho_2 (k_1 h_2 - k_2) \quad (3.59)$$

(b) REHEATER STEAM

$$\frac{d W_{PR}'}{dt} = \frac{W_{PR} - W_{PR}^1}{T_{R1}} \quad (3.60)$$

$$\frac{d Q_R}{dt} = \frac{(W_{PR} + W_{PR}^1) H_R}{2 T_{R2}} (T_S - T_R) - \frac{Q_R}{T_{R2}} \quad (3.61)$$

$$W_{PR} = C_2^* A_2 P \quad (3.62)$$

$$T_R = \frac{P_R}{R \rho_R} \quad (3.63)$$

## IP and LP TURBINE

$$\frac{d W_3'}{dt} = \frac{1}{T W_3} \left[ (1 - K_{BLP}) W_3 - W_3' \right] \quad (3.64)$$

$$W_{BLP} = K_{BLP} W_3 \quad (3.65)$$

## HEATER 1

$$\frac{d h_{fw}^1}{dt} = \frac{Q_{H1}}{T_{H1} W_{FW}} + \frac{h_o^1 - h_{fw}^1}{T_{H1}} \quad (3.66)$$

$$Q_{H1} = H_{FW} (W_{HP}^1 + W_{BLP}) \quad (3.67)$$

## HEATER 2

$$\frac{d h_{fw}}{dt} = \frac{Q_{H2}}{T_{H2} W_{FW}} + \frac{h_{fw}^1 - h_{fw}}{T_{H2}} \quad (3.68)$$

$$Q_{H2} = H_{FW} (W_{BHP} + W_{MS} + W_{PR}^1) \quad (3.69)$$

$$\frac{d W_{HP}^1}{dt} = \frac{W_2 - W_2'}{T_{H2P}} + \frac{W_{PR}^1 - W_{HP}^1}{T_{H2P}} \quad (3.70)$$

## OUTPUT EQUATIONS

$$T_{HP} = K_T \eta_{HP} W_2 (h_C - h_2') \quad (3.71)$$

$$T_{LP} = K_T \eta_{LP} W_3'' (h_R - h_4') \quad (3.72)$$

$$\eta_{HP} = \eta_{CFHP} \eta_{HP}^* \quad (3.73)$$

$$\eta_{LP} = \eta_{CFLP} \eta_{LP}^* \quad (3.74)$$

$$\eta_{LP}^* = \eta_{HP}^* = \frac{h_S - h_2}{h_S - h_2'} = 0.86 \quad (3.75)$$

$$\eta_{CFHP} = \frac{W_2' / W_2^* - W_L}{W_2 / W_2^* (1 - W_L)} \quad (3.76)$$

$$\eta_{CFLP} = \frac{W_3' / W_3^* - W_L}{(W_3' / W_3)(1 - W_L)} \quad (3.77)$$



$$W_L = 0.02 \quad (3.78)$$

$$\text{TORQUE} = T_{HP} + T_{LP} \quad (3.79)$$

$$\text{POWER} = \text{TORQUE} \times \Omega \quad (3.80)$$

#### THE PARAMETERS IN EMPIRICAL FORM

$$V_f = 0.0184 \quad (3.81)$$

$$V_g = 2.288 - 0.0166 (P_R - 200) \quad (3.82)$$

$$h_S = [1190 - 0.0256 (P - 1000)] 1.055 \times 10^3 \quad (3.83)$$

$$h_f = [355.6 + 0.44 (P_R - 200)] 1.055 \times 10^3 \quad (3.84)$$

$$h_{fg} = [843 - 0.4 (P_R - 200)] 1.055 \times 10^3 \quad (3.85)$$

$$h_g = h_f + h_{fg} \quad (3.86)$$

$$H_2' = (1067 + .37 (P_R - 200) - 0.0011 (P_R - 200)^2 - 0.1 (P_C - 1000) 1.055 \times 10^3) \quad (3.87)$$

$$h_o = 69.0 \times 1.055 \times 10^3 \quad (3.88)$$

$$R = \frac{208}{0.372856 (520 + 460)} \quad (3.89)$$

$$h_4' = 898 * 1.055 * 10^3 \quad (3.90)$$

#### 3.4 MODIFICATIONS INTRODUCED

The non-linear models for sub-assemblies viz. reactor and turbine, presented above in Sec. 3.3.1, Sec. 3.3.2 were linearised around an operating point and validated for small order transients in earlier studies Refs. 11, 12, 13, 15 . The present interest is to

validate and study the non-linear models for the entire assembly for large order transients, viz., RC pump failure and turbine trip etc. In order to do this, the two sub-assemblies are coupled together and the simulation is carried out for the RC pump trip failure test. It is found that the response for neutron power is very sudden and the absolute values of various variables are far from satisfactory (Fig.3.2). So it is felt that the model needed modifications and the following describes the effort in this direction. These modifications are (a) better throttle valve representation (b) incorporation of the pump dynamics and (c) inclusion of sweep effects of voids. This study was presented in a recent paper by GIRIJASHANKAR, CHAWLA and MALHOTRA [57], where the improved effects were exclusively shown. These improvements are described as follows.

#### 3.4.1 THROTTLE VALVE

Isenthalpic throttle valve connecting the reactor to HP turbine was represented by a linear relationship between steam flow, area of flow and upstream pressure in Sec. 3.3.2 by Eqn. 3.47. This is valid only if the downstream pressure is kept constant and the valve movement is small in the neighbourhood of an operating point. However, in the large transient study, since the reactor feeds steam to the turbine the downstream pressure is likely to change and the valve movement is large enough

to introduce nonlinearities in the flow, pressure and valve movement characteristics. Thus, the steam flow through the valve is a nonlinear function of the valve-stem position and difference of upstream and downstream pressure rather than upstream pressure. This new relationship is used while representing the valve, in accordance to IBM report, 1972, as shown below

$$W_1 = C_{VH} * (y_0 - y)^N * (p_S - p_C) \quad (3.91)$$

The constant  $C_{VH}$  is determined under the initialisation conditions, KEARTON [55]. The value of  $N$  is selected such that at rated flow, a ten percent closing of the valve stem yields an approximate ten percent reduction in the flow rate. This equi-percentage valve characteristic is obtained by  $N = 2$ . This (i.e. Eq. 3.91) essentially simulates the valve for large valve-stem lifts and pressure difference changes.

### 3.4.2 PUMP DYNAMICS

While simulating the pump trip of the combined model of the reactor and turbine, earlier, the term POWP in Eqn. 3.11 was set to zero. The neutron power response was compared with the available test result (Fig.3.2 ). Two major differences were observed (a) the dip in the neutron power appeared quite early i.e. at less than one second, where as in the actual plant it occurred at

nearly four seconds and (b) the magnitude of the dip was much smaller. It was thought that this was mainly due to ignoring the pump dynamics. The pump failure was represented as loss of power to the pump (i.e. POWP = 0). This actually means that the pump pressure developed by the pump reduces to zero immediately. This however will not happen in the actual pump. In an actual pump, flow settles at 10% of the value in say 5 seconds depending on pump inertia instead of much faster coastdown as in the earlier case. This has strong influence on the neutron power variation. Also, it is well-established fact that the direct-cycle BWR power plant can be controlled through changes in recirculation pump speed, JOSEFSSON [59]. The above two reasons prompted the modelling of the pump dynamics. In boiling water reactor the recirculation pumps are driven by induction motors. Thus, the pump speed change is a function of the impressed electrical torque and the mechanical torque developed by the pump. Applying Newton's second law of motion to the rotor, the pump dynamics is represented by LINFORD [16]

$$\frac{dn_p}{dt} = \frac{30 g_C}{\pi J_p} (T_{el} - T_m) \quad (3.92)$$

where  $J_p$  is the combined moment of inertia of motor, impeller of the pump and other rotating parts. The mechanical torque depends upon the pump speed ' $n_p$ ', and the flow rate  $W_D$  as shown in Fig. 3.3 Disregarding

small motor time constants, the electrical motor torque is obtained from the torque-speed characteristics of the motor.

These functional relationships can be represented by

$$T_m = f_1 (W_D, n_p) \quad (3.93)$$

$$T_{el} = f_2 (n_p) \quad (3.94)$$

The graph for  $T_{el}$  vs  $n_p$  has not been considered since  $T_{el}$  immediately goes to zero with the loss of power. The head developed by the pump itself is a function of the pump speed  $n_p$  and the flow rate  $W_D$  and is given by

$$\Delta P_P = f_3 (W_D, n_p) \quad (3.95)$$

Such a characteristic is given in Fig. 3.4. Thus, the Eqn. 3.11 can be replaced by Eqn. 3.95. For simulation purposes, the pump characteristics of Fig. 3.3 and 3.4 are resolved into a mesh of  $T_m$ ,  $\Delta P_P$  values for flow rate with speed as a parameter. These curves are approximated by choosing a small enough interval size and the intermediate values are calculated by a linear interpolation technique.

### 3.4.3 SWEEP EFFECTS

In the reactor sub-system, the core average void fraction,  $\bar{X}_C$ , and the average quality,  $\bar{X}_C$ , were obtained

even during transients by applying quasi-static heat balance equations to the core. Thus Eqns. 3.21 and 3.23 were algebraic in nature and did not take into account the dynamic dependence of voids on flow rate and pressure changes. ACKASU [60] was first to point out the need for representing the dynamic behaviour. This behaviour has an important role to play in transients where large amount of flow rate and pressure variations occur e.g., RC pump failure. As soon as power to the pump is lost, a rapid drop in the flow rate occurs in the entire primary loop including the core. This is accompanied with the reduction in heat removal rate from the core and results in the void build-up. The neutron power due to reduction in thermalisation (-ve void coeff. of reactivity), tends to drop. The model used earlier did take into account the above features. However, with the drop in the coolant velocity, the voids generated earlier were not being swept as quickly as with the full velocity case. The sweep phenomenon can be understood with the help of the following arguments. Let us watch the moment of the bubbles in a heated channel shown in Fig.3.5 . It is assumed that the generation in each section is lumped and is the same (generation rate is constant) i.e. two bubbles per section Fig. (3.5 a) shows the void distribution in an initial steady state before the pump fails. Now that the velocity drops down to half its original value. The sequence of the events at

every time step is depicted in Fig. (3.5 b) to Fig. (3.5g). In constituting this the bubble generation rate is assumed to be the same as earlier i.e. two bubbles per section. This is a simplifying assumption and may not be valid in practice. In the figures '0' shows the bubble generated at a particular instant of time while '●' shows the bubble generated at earlier time but which has moved up. In each of the subsequent figures, the bubble marked 1 represents the bubble generated at the zeroth step i.e. Fig. (3.5 b). It is seen that this bubble marked 1 moves out of the channel in six time steps. Further, the bubble pattern in this channel from the sixth time step onwards is same as in fifth time step and the new steady state is reached. Since the channel was divided into 3 sections it takes six time steps for the bubble marked 1 to escape from the channel i.e. it takes the new transit time (calculated on the basis of  $V_1/2$  and core length) for the bubbles to stabilize at the new steady state. In observing the Fig. (3.5 ), it is clear that the maximum bubble accumulated is in the 3rd, 4th and 5th time step. This is known as the sweep effect.

We will represent the dynamics of the void sweep phenomenon by a first order and a single time constant dynamics. Thus

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$$\frac{d \alpha_s}{dt} = \frac{2}{T_{BS}} ( \alpha_C + K \alpha_1 (W_D(t) - W_D(t - T_{BS})) - \alpha_s ) \quad (3.96)$$

$$\begin{aligned} T_{BS} &= \text{average transport time across the boiling} \\ &\quad \text{length, secs.} \\ &= \left( \frac{\text{boiling length}}{\text{core inlet velocity}} \right) \end{aligned} \quad (3.97)$$

The coefficient  $K\alpha_1$  is the partial derivative of void fraction  $\alpha_C$ , with respect to flow rate,  $\text{ft}^3/\text{sec.}$ , assumed constant and obtainable from a steady-state void map around the given operational point. The occurrence of  $T_{BS}$  in the term  $W_D(t - T_{BS})$  in Eqn. 3.96 indicates that the sweep effect is felt for a period corresponding to the effective passage time for the flow across the boiling length in the core.

The need of sweep effects was strongly felt from the study of the results presented in Ref. [11], where, the void reactivity coefficient was increased five times to a pseudo value of  $\frac{115}{2} \text{C}/\%$  to obtain better agreement for the pump trip tests. This showed that not all the voids have been accounted for in the steady state void representation, whose feedback effect on neutron power is most dominant in the present test. Both the 'pseudo' void coefficient approach and the explicit modelling of sweep have been carried out presently, the latter being



seen to give quite meaningful comparisons with the experimental results.

#### 3.4.4 COUPLING OF THE TWO SUB-SYSTEMS

The modified model for the sub-systems i.e., the reactor and the turbine are coupled to represent the complete plant. This is made possible in the model by the following features. In the physical process of Fig. 3.1 the steam drum supplies steam of 99.9% quality in the main pipe line, a part of which drives the HP turbine and the rest is supplied to the shell and tube type reheater for superheating the steam coming out of the HP turbine before feeding it to the IP and LP turbine. This is represented by replacing Eq. (3.36) by

$$W_S = W_1 + C_2^* A_2 P \quad (3.99)$$

This steam traverses the turbine loop and comes out of Heater 2 with certain enthalpy  $h_{FW}$ . The Eqn. 3.27 assumed a constant feedwater temperature which, however, keeps varying due to the turbine dynamics as seen in Eqn. 3.68. Thus the old Eqn. 3.27 is replaced by the following representation

$$T_m - T_S = - (h_F - h_{FW}) * \frac{W_{FW}}{W_D C_f C_p} \quad (3.100)$$

This feedwater, returned from the Heater 2 is fed into

steam drum where it mixes with the recirculation water which is at saturated conditions. Since these two are at different temperatures, mixing process in the drum returns the mixture at a mixing temperature to the core through down-comer. This mixing is not modelled. However, the delay of 6 seconds associated in the mixing process is modelled as a pure transport delay in the simulation. Thus the total delay is 15 secs. instead of 9 secs. used earlier. The new plant model used for simulation, consists of 21 differential equations and 75 algebraic equations. The differential equations are 3.1, 3.2, 3.7 to 3.10, 3.28, 3.34, 3.45, 3.46, 3.52, 3.56, 3.57, 3.60, 3.61, 3.64, 3.66, 3.68, 3.70, 3.92 and 3.96.

$$\text{Defining } \underline{X}^T = \left[ N, C, T_F, Q, V_{IN}, T_I, P, W_{FW}, h_C, t_C, \right. \\ \left. W_2'', \phi_R, h_R, W_{PR}', Q_R, W_3', h_{fw}', h_{fw}, \eta_p, \right. \\ \left. \alpha_S \right] \quad (3.101)$$

$$\text{and } \underline{U}^T = \left[ \Delta k_{ex}, T_{el}, y, A_2 \right] \quad (3.102)$$

It can be seen that the structures of the equations given above are of the type

$$\dot{\underline{X}} = \underline{F} \left[ \underline{X}, \underline{U}, \underline{Z}, \underline{X}(t - \tau) \right] \quad (3.103)$$

$$\underline{Z}^T = \phi \left[ \alpha_x, \alpha_R, x_{ex} \right] \quad (3.104)$$

where  $\underline{X}$  and  $\underline{U}$  are state and control vectors.

Closer look at the equations would reveal that the terms in  $\underline{Z}$  can be expressed explicitly in terms of  $\underline{X}$  and  $\underline{U}$ , i.e.

$$\underline{Z} = g(\underline{X}, \underline{U}) \quad (3.105)$$

Therefore substituting eqn. 3.105 in 3.103 will yield nonlinear differential difference equations in the form of

$$\dot{\underline{X}} = f(\underline{X}, \underline{X}(t - \tau), \underline{U}) \quad (3.106)$$

Simulation of the system equations will be discussed in next section.

### 3.5 SIMULATION OF THE COMPLETE PLANT

The plant model was simulated by Runge-Kutta 4th order technique, according to GILL [61]. The IBM 7044 computer took 2.7 secs. for simulating one sec. of the non-linear plant model. In order to reduce the computer time an approximation was used in the model. This will be discussed in Sec. 3.5.2.

#### 3.5.1 VALIDATION OF THE MODEL

The plant model was initialized to yield a corresponding plant operations data before any simulation test was conducted. For simulation purposes one of the variable operating pressure was selected around 1000 psi, since the empirical fits for enthalpy, saturation temperature etc. were made in the vicinity of 1000 psi to represent Mollier chart and steam table parameters used

for calculation. For initialisation the input parameters to the reactor e.g. valve lift, motor electrical torque was selected from the data while all other parameters were calculated from the steady state conditions by equating the differential equations to zero. One such operating point obtained is given in Table 1. Also the corresponding operating data from the plant is shown. The close correspondence between the two can be observed. The plant model is validated only for the Pump Trip, and the Control Rod Oscillation tests because of the limited availability of the experimental results. Then the following studies are also conducted.

### 3.5.2 OTHER STUDIES

#### TRIAL OF PROMPT JUMP APPROXIMATION

When Eq. 3.1 for the Neutron kinetics is used, the integration step size is limited to 0.005 as any larger step size would introduce integration error. In order to overcome this prompt-jump approximation was tried to reduce computer time taken for the calculations. The above approximation is valid when the relative rate of change of reactor power  $N$  in Eq. 3.1, the mean prompt generation time  $\lambda$ , is sufficiently small so that

$$\frac{1}{\beta} \left( \frac{dN}{dt} \right) \frac{1}{N} \ll \left| 1 - \frac{\Delta k \lambda'}{\beta} \right| \quad \text{holds, such that the}$$

term  $\frac{\lambda}{\beta} \frac{dN}{dt}$  in the kinetic Eqn. 3.1 can be neglected in comparison to  $(\Delta k \frac{\lambda}{\beta} - 1) N(t)$ . This approximation is called prompt jump approximation because it predicts the sudden change in the reactor power following a sudden change in reactivity when the concentration of precursors remain constant and are represented by one group pressure model. So Eqn. 3.1 was replaced by linear algebraic equation

$$N = \frac{\lambda}{\beta - \Delta k} \lambda c \quad (3.107)$$

This is described in detail by AKASU [62]. This PJ approximation was first tried to reactor subsystem using 0.5 sec. time step. Thus 100 times faster computer calculations were possible (1 computer sec. = 38 problem sec.), introducing less than a maximum of 3% absolute error. Figure 3.6 shows the trends for various time steps for large transient like pump trip using the PJ approximation. But this PJ approximation was not used in the coupled system as the integration step size used for the turbine part was the limiting factor as compared to the reactor part.

TABLE No. 1

## EXPERIMENTAL AND COMPUTED PLANT DATA

PARAMETER	COMPUTED	EXPERIMENTAL MEASUREMENTS
1. Neutron Power, Watts	0.147166 E 09	0.149 E 09
2. Delayed Neutrons, "	0.200235 E 11	
3. Average Fuel temp., °F	0.788573 E 03	
4. CORE Inlet Velocity, ft/sec	0.56833 E 01	
5. CORE Inlet Temp. °F	0.541747 E 03	0.539 E 03
6. Reactor Vessel Pressure psia	0.104384 E 04	0.1046 E 04
7. Feed water weight flow, lbm/sec	0.159742 E 03	
8. Steam weight flow, lbm/sec	0.15973 E 03	
9. Recirculation flow rate, ft <sup>3</sup> /sec.	0.772930 E 02	0.7816 E 02
10. Enthalpy at inlet to HP turbine W Sec	0.1254207 E 02	
11. Flow at outlet of HP turbine lbs/sec	0.1423000 E 03	
12. Density of steam in the nozzle lbs/m <sup>3</sup>	0.2325000 E 01	
13. Density of steam at reheater exit lbs/m <sup>3</sup>	0.3728000 E 00	
14. Enthalpy of steam at reheater exit W sec.	0.1350000 E 06	
15. Reheating water at reheater exit lb/sec.	0.1149000 E 02	
16. Heat transferred in the reheater W	0.1019000 E 08	
17. Flow of steam at LP turbine exit lb/sec.	0.9200000 E 02	
18. Enthalpy of water at re- heater 2 entrance W Sec	0.2357000 E 06	

Continued

PARAMETER	COMPUTED	EXPERIMENTAL MEASUREMENTS
19. Enthalpy of water at Reheater 2 exit W sec	0.3245000 E 06	
20. Cumulative bleed to Reheater 2 lb/sec	0.3687700 E 02	
21. Core average exit quality	0.0593120 E 00	0.045
22. Core average exit void fraction	0.5124500	0.45

b. Other tests that have been simulated on the model are

- i) 20% and 80% step change in the main throttle valve,
- ii) valve rod oscillation,
- iii) turbine trip .

### 3.5.3 CONCLUSION

The model for the BRP boiling water reactor system was presented with a critical approach introducing the modifications for better representation of the plant. These modifications included throttle valve better representation, the incorporation of pump dynamics and the sweep effects. This modified plant was used for simulation purpose. The procedure for simulating the pump failure test was discussed. PJ approximation to minimize the computer time was next discussed.



FIG. 3.1 BLOCK DIAGRAM OF THE BRP SYSTEM



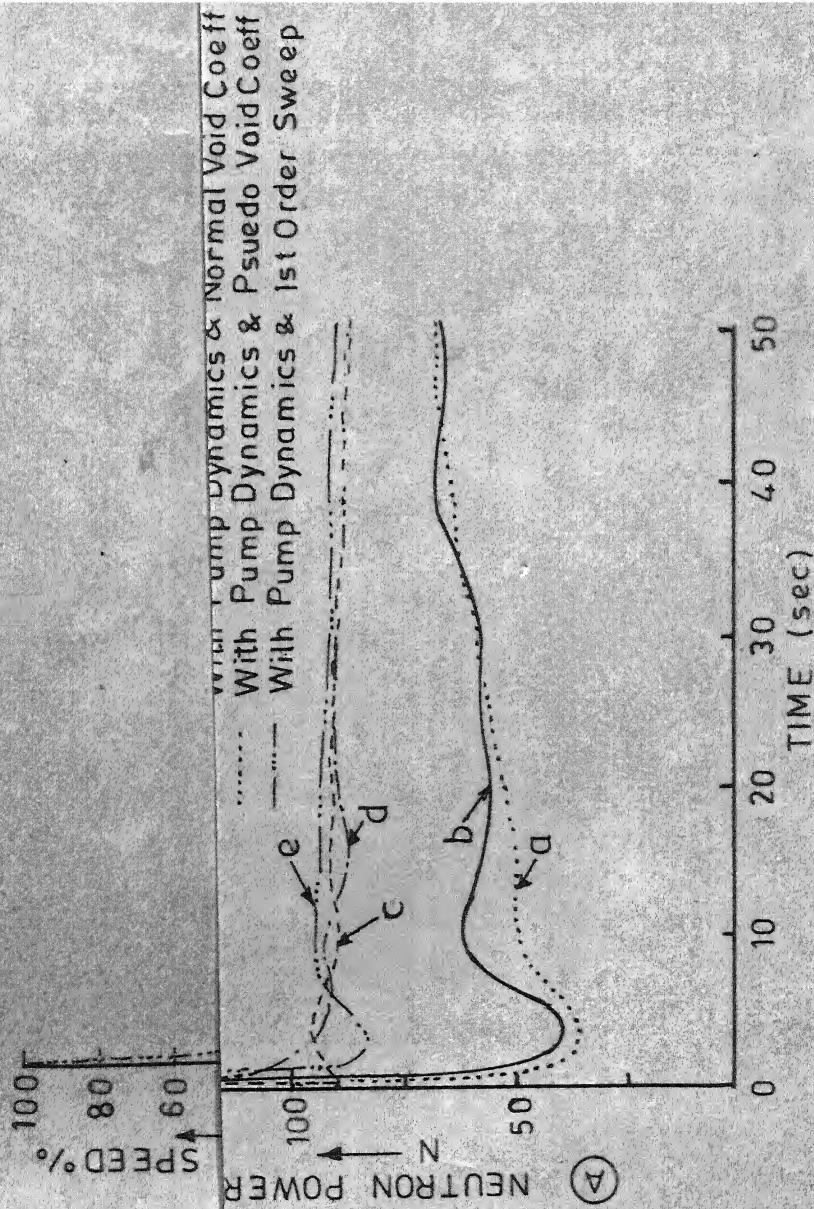


FIG.3.2 COMPARISON OF SIMULATION AND PLANT  
 TEST RESULTS FOR RC PUMP FAILURE

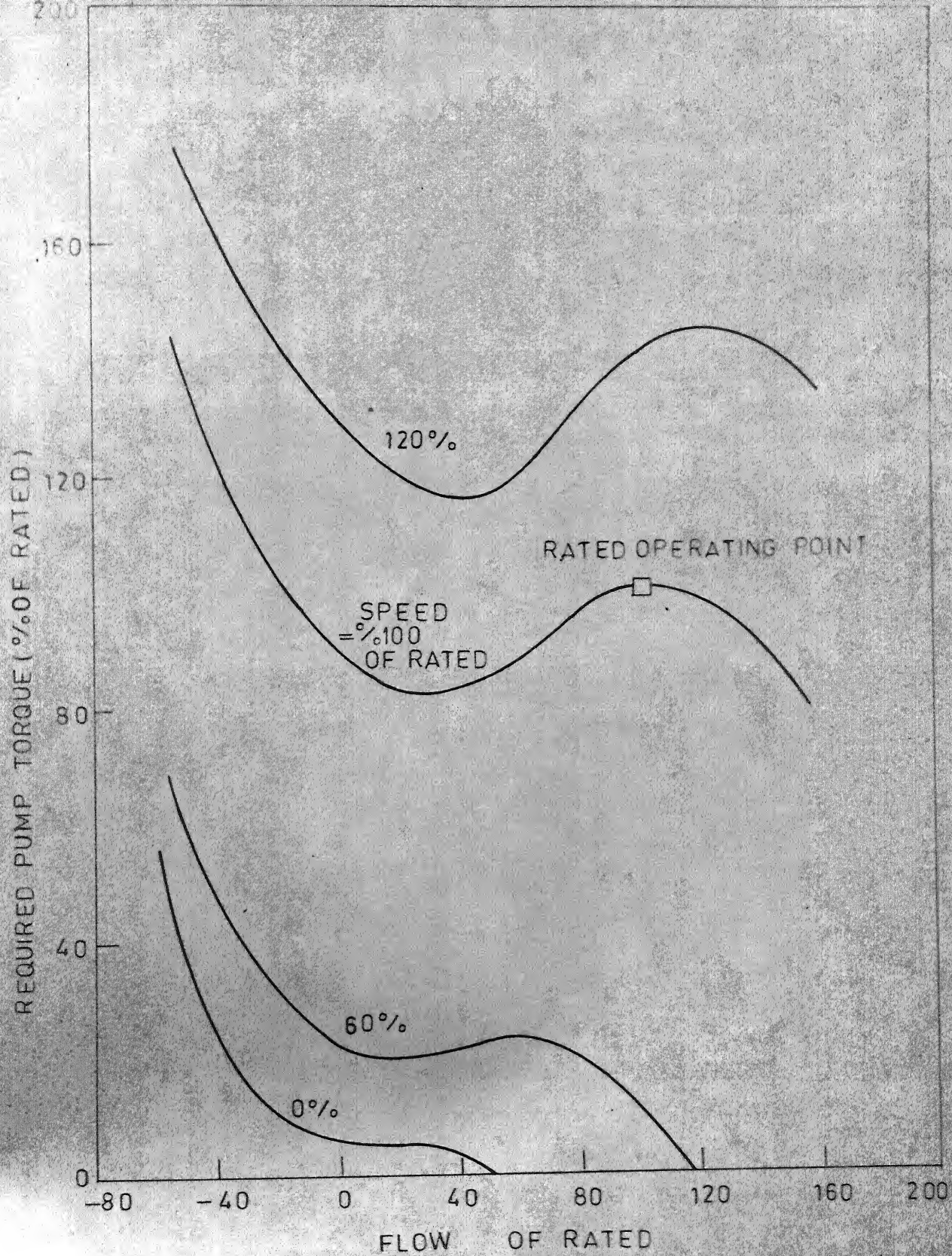


FIG. 3.3 TYPICAL PUMP TORQUE CHARACTERISTIC VERSUS FLOW AND SPEED



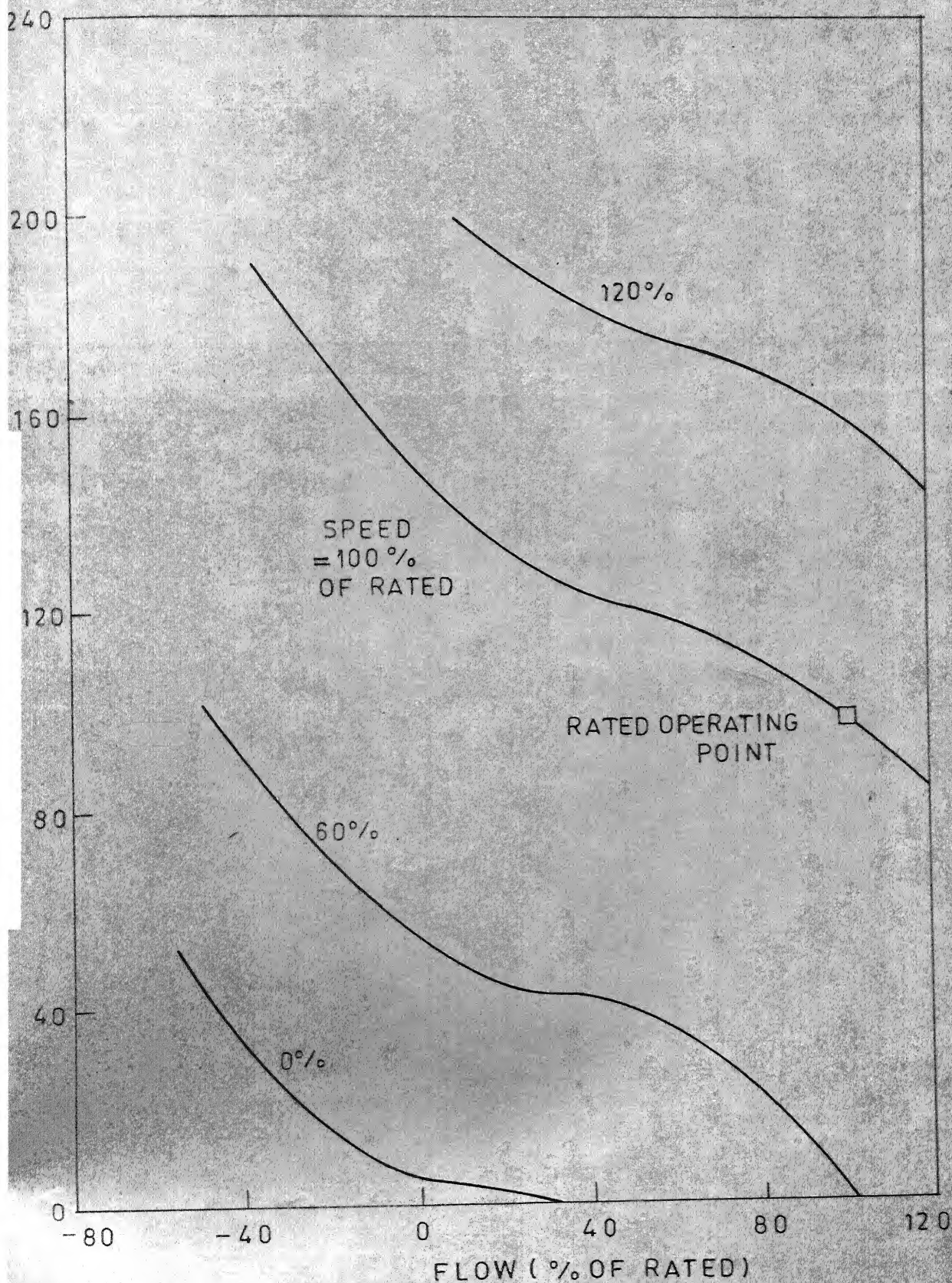


FIG. 3.4 TYPICAL RECIRCULATION PUMP HEAD VERSUS FLOW AND SPEED

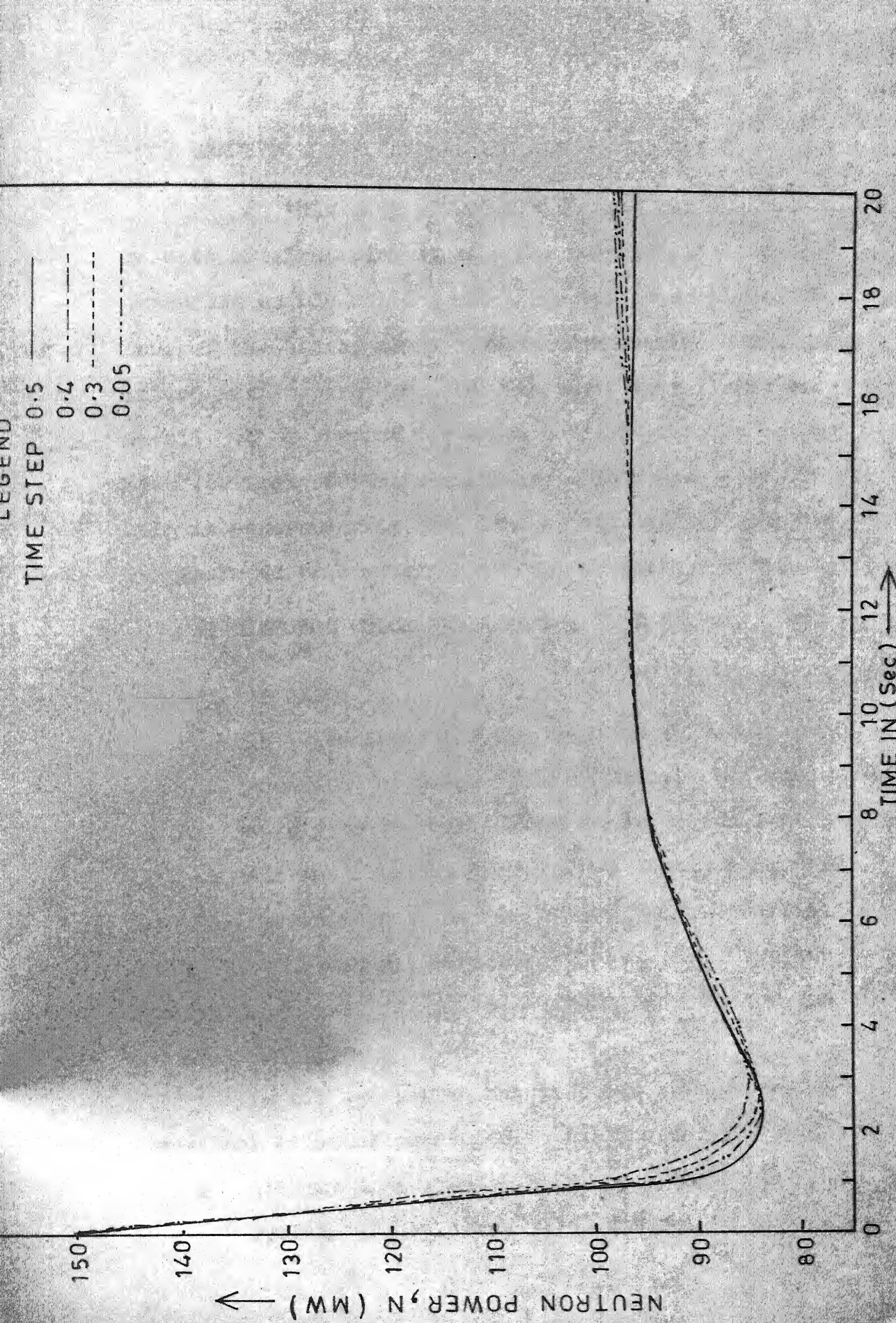


FIG. 3.6 NEUTRON POWER (MW) TRANSIENT AT DIFFERENT COMPUTATIONAL TIME STEPS WHEN TWO PUMPS TRIP

## RESULTS, CONCLUSIONS AND RECOMMENDATIONS

In this chapter, we present and discuss the results of simulation studies for some selected tests conducted on BRP boiling water reactor power plant. For some of the tests, plant results were available and simulation results are compared with those to validate the model. Other results for which comparison could not be made for lack of test results are also presented as they help in understanding the plant behaviour. The chapter is concluded with recommendations for further work.

## 4.1 STUDIES FOR WHICH EXPERIMENTAL TEST RESULTS WERE AVAILABLE

## 4.1.1 PUMP TRIP

It is conducted on the plant by switching off the electric power to the pump. This results in the pump speed to fall rapidly to the value where it is driven as a turbine. This is made possible due to the sufficiently available NPSH (net positive suction head) thus the reactor will operate on the natural circulation mode. This enables the reactor to deliver 50% of the nominal power at the end of the transient.

In the simulation the test was implemented on 4 versions of model presented in the Chapter 3. They are

- i) without inclusion of the pump dynamics the term POWP in  $\Delta P_p$  (Eqn. 3.11) was set to zero,



- ii) the pump dynamics was incorporated and the term  $T_{el}$  in Eqn. 3.92 was equated to zero, using normal void coefficient of reactivity (11.5  $\beta/\%$ ),
- iii) using pump dynamics and pseudo void-coefficient of reactivity (5 times the normal value, refer to IBM report [11]), the term  $T_{el}$  in Eqn. (3.92) equated to zero,
- iv) the pump dynamics and sweep model was incorporated equating  $T_{el}$  to zero in usual manner.

Figure 3.2 shows the response<sup>for</sup> (A) the neutron power, (B) the average heat flux, (C) the recirculation flow by all above mentioned four methods. However, the performance for (D) the fuel average temperature and (E) the pump speed presented for only the (iv) case, because other methods revealed similar trends. First the plant results are discussed followed by the simulation results. In Fig. (3.2b) for the neutron power the initial dip is observed at nearly 4 seconds then it tries to stabilise at 75 MW<sub>t</sub> after 50 seconds (Fig.3.2b). The recirculating coolant flow in the core  $W_D$  decreases (Fig. 3.2k) resulting in lesser heat removal from the fuel assembly. This, essentially, reduces the heat transfer rate  $Q$ , although at a slower rate.

Thus, the average core quality  $x_c$  increases (Eqns. 3.18 and 3.19) increasing the amount of voids in the core ( $\alpha_{ex}$ , Eqn. 3.21). The increase in voids reduces the thermalisation action of the moderator in Eqn. 3.1 resulting in a reduction of  $\Delta k$  the net reactivity of the core through  $\Delta k_\alpha$  (Eq. 3.3). This sequence of events continues till 6 seconds or so and the dip in the neutron power is noticed.  $T_F$  through Eqn. 3.7 also decreases (Fig. 3.2 p) resulting in the reduced neutron absorption in the fuel due to Doppler effect. The net effect of  $\Delta k_{DOP}$  and  $\Delta k_\alpha$  is that they try to annul the effect of each other, till one exactly balances the other and a new steady state for the neutron power is reached (Fig. 3.2 ).

The simulation results also reveal the same nature of the test results given above. The Fig. 3.2 for the neutron power clearly brings out the improvements on implementing the four simulation efforts.

The dip (Fig. 3.2 c) occurs much earlier (1 sec.) as compared to 4 seconds in the test result (Fig. 3.2b). The incorporation of the pump dynamics, (Fig. 3.2d) (ii) improves the situation with regard to time behaviour, (Fig. 3.2 d) while inclusion of the first-order sweep effect (iii) gives better magnitude of the dip (Fig. 3.2 e) The fourth method, using pseudo void coefficient, interestingly, shows an excellent correspondence of the model

with the plant for the neutron power. This reveals strong influence of the voids on the reactor behaviour indicating the inadequacy of the first - order sweep. Case (ii) and (iii) individually, overestimates while (iv) under-estimates the time response for the heat flux. In the actual plant also the heat transfer rate is measured using the neutron power as an input to the analogue integration scheme employing operation amplifiers etc. with a time constant of 6.6 seconds as has been done in the model here. Thus the above trend for the four different cases can be explained due to this. Since the reactor pressure was held constant with the help of I.P.R. this transient is not felt on the turbine side of the plant at all. For other studies reported here, the normal void coefficient of 11.5  $\text{¢}/\%$  is used. Although, the use of pseudo void coefficient exhibited excellent correspondence of model with the plant, yet there was no strong justification for it to be used for other tests. Moreover, during other studies, recirculation coolant flow undergoes smaller change and hence the effect of void changes due to flow changes is negligible. Thus the inclusion of the normal void coefficient does not introduce significant errors it was used in other tests.



#### 4.1.2 CONTROL ROD OSCILLATION

The control rod oscillation is a steady state test, normally conducted on plant to measure the reactivity-to-neutron flux transfer function by oscillating a standard control rod in a certain frequency-range. The steam pressure controller, the water level controller and the other control rods are kept on manual to avoid variation of pressure and temperature due to changes in these control inputs. In the simulation this test was conducted by varying the  $\Delta k_{ex}$  term by 2  $\%$  in Eqn. 3.3 sinusoidally, at a frequency of one cycle per sec. (Fig. 1a). The neutron power is compared with the available BRP plant result, under same conditions of magnitude and frequency of the control rod variation. The comparison of the simulated and the plant test result for the neutron power is shown in Fig. 4.1c.

It is observed that:

1. the neutron power of the plant and the simulation results in Figures 4.1c and 4.1b varies with same frequency at which the control rod oscillates (1 Cps),
2. the nature of the variation of neutron power (comparing the graphs a and b plotted on same scale) is similar for both the plant and the simulation,
3. There is however a small phase difference of about  $8 - 10^\circ$  between the plant and the simulated results.

A maximum amplitude of 0.5 MW increase in  $Q$  was observed which is negligibly small as compared to 7 MW change in the neutron power and also  $T_F$  is changed by a maximum amplitude of  $0.8^{\circ}\text{F}$  (not shown in the figure). This perturbation on the plant is however, damped out before it reaches the turbine.

## 4.2 OTHER STUDIES

### 4.2.1 20% AND 80% STEP INSERTION OF VALVE STEM IN THE THROTTLE VALVE

The throttle valve interconnects the reactor sub-assembly with the turbine sub-assembly. In practice, the step changes on the valve stem are introduced to study its influence on both the reactor and the turbine.

In the simulation it was performed by setting  $y$  to  $0.20 y_0$  and  $0.80 y_0$  respectively in the equation (3.99) of the following term which represents the throttle valve

$$W_1 = C_{VH} * (y_0 - y)^2 * (P_S - P_C) \quad (4.1)$$

The responses for these disturbances are shown in Fig. 4.2 . The comparison with the test result could not be done due to the non-availability of the test data. The two disturbances of different magnitude carried out exhibit similar trends for the plant variables. It is

clear from Fig 4.2 that 80% step change has a stronger influence on the plant as is to be expected and it is interesting to study its influence on the plant. As soon as the valve stem is dropped it reduces the area of steam flow.

The steam flowing out of the steam drum reduces immediately and the dip occurs after 0.5 seconds (Fig. 4.2a ). This causes the steam flowing in the steam chest to reduce and

$f_C$  and  $h_C$  as per equations 3.45 and 3.46 suddenly decrease. These result in quick fall of the steam chest pressure ( $P_C$ ) as per equation 3.50 with the dip in  $P_C$  occurring at 0.5 sec. (Fig. 4.2b). The lowering of the steam drawn from the drum is felt by the reactor also which builds up the steam pressure through equation 3.28. Thus the saturation temperature of the steam increases (equation 3.40) resulting in increase of  $Q_{SUB}$  (equation 3.29) while (equation 3.18) decreases. This, effectively suppresses the voids in the core (equation 3.21 and 3.23) and the neutron thermalisation improves suddenly causing a quick jump in  $N$  at 0.5 sec. (Fig. 4.2d). Simultaneously, on the turbine side the steam flow to the HP unit decreases resulting in the decrease of  $f_R$ ,  $h_R$  (equations 3.56 and 3.57) which through equations 3.58 and 3.63 decrease  $P_R$  and  $T_R$ . The response for  $T_R$  is shown in Fig 4.2c . The steam flow passing to the reheater,  $W_{PR}$  shows a small increase due to the increase in pressure  $P$ . This increased steam flow rate

and the decrease in  $T_R$  mentioned earlier results in an increase in  $Q_R$  as per equation 3.61 (Fig. 4.2f ). In equation 3.71 both  $h_s$  and  $h_2'$  increase, the first quantity, because of linear dependance on pressure  $P$  (equation (3.83)) and the latter due to its dependance on  $P_R$  (equation 3.87) but the  $h_2'$  increase is much more compared to  $h_s$  increase so as to decrease  $(h_s - h_2')$ . Also  $W_2$  decreases (not shown in Fig. 4.2 ). These result in  $T_{HP}$  to decrease (Fig. 4.2g). The variable  $h_R$  decreases gradually so that  $T_{LP}$  (Eqn. 3.42) pursues a gradual decrease (Fig. 4.2h ). The reactor pressure  $P$  keeps building up suppressing the voids and hence neutron power increases till the average fuel temperature effect is substantial i.e., the negative feed back effect due to Doppler effect of reactivity. The neutron power continues to build-up slowly till 15 seconds when the mixing water from the drum flows back to the inlet of the core with an increased sub-cooled enthalpy. This causes the voids to increase and hence it forces neutron power to decrease. Thus from this stage onwards, the net effect on the neutron power will be due to these three i.e.

- i) flashing effect due to the pressure build-up  
(i.e. void collapse due to the pressure rise),
- ii) increased void generation rate due to decreased subcooling,
- iii) Doppler effect.

The net effect results in decreased neutron power.

The temperature  $T_R$  decreases because of the valve closure (this is due to the decrease in  $P_C$  also), although the flow rate passing through the secondary steam valve is not changed in this simulation. The decrease in temperature is observed as shown in Fig. 4.2e. The combined effect of the temperature  $T_R$  decrease and the main steam flow rate decrease is to increase the heat transfer rate as per equation (3.61) and is shown in Fig. 4.2f. The response of the plant is simulated till 20 seconds. It is seen that the throttle valve serves effectively to control the turbine output. Fig. 4.2j shows the decreasing trend of the mechanical power produced.

#### 4.2.2 THROTTLE VALVE ROD OSCILLATION

This steady state test is performed to know the variation of the pressure, the neutron power and the mechanical power with the variations in the steam flow rate.

In the simulation, it was conducted by sinusoidally oscillating the valve set point at a frequency of 1.57 Cps and a magnitude of 40% of relative position through equation 3.91. The responses of the following:

- A. the relative valve movement,

- B. the steam flow rate  $W_S$  ,
- C. the vessel pressure,
- D. the neutron power,
- E. the heat flux,
- F. the mechanical power are shown in Fig. 4.3e.

The interactions between various variables has already been discussed in Sec. 4.2.1.

There is an initial transient especially in the neutron power Fig. 4.3d which however dies down in the first 3 seconds and afterwards reaches steady state oscillation with about 1/2 MW magnitude. While both neutron power and reactor pressure Fig. 4.3 are nearly in phase with the valve rod oscillation, both mechanical power (Fig. 4.3c & e) and steam flow rate (Fig. 4.3b) have  $\sim 180$  degree phase difference with valve rod oscillation. Also a flattening of both steam flow rate and mechanical power on the peak side of the cycle is observed. This effect for steam flow is due to the nonlinear valve admittance,  $C_{VH} (y_0 - y)^2$ , versus flow  $W_1$  characteristics given by Eq. 3.91 and thus admittance leads to saturation of the flow rate at high flow rates. Since mechanical power directly depends on flow rate this flattening is also observed in the mechanical power curve (Fig. 4.3e)

- B. the steam flow rate  $W_s$  ,
- C. the vessel pressure,
- D. the neutron power,
- E. the heat flux,
- F. the mechanical power are shown in Fig. 4.3e .

The interactions between various variables has already been discussed in Sec. 4.2.1.

There is an initial transient especially in the neutron power Fig. 4.3d which however dies down in the first 3 seconds and afterwards reaches steady state oscillation with about 1/2 MW magnitude. While both neutron power and reactor pressure Fig. 4.3 are nearly in phase with the valve rod oscillation, both mechanical power (Fig. 4.3c & e) and steam flow rate (Fig. 4.3b ) have  $\sim 180$  degree phase difference with valve rod oscillation. Also a flattening of both steam flow rate and mechanical power on the peak side of the cycle is observed. This effect for steam flow is due to the nonlinear valve admittance,  $C_{VH} (y_0 - y)^2$ , versus flow  $W_1$  characteristics given by Eq. 3.91 and thus admittance leads to saturation of the flow rate at high flow rates. Since mechanical power directly depends on flow rate this flattening is also observed in the mechanical power curve (Fig. 4.3e)

### 4.2.3 TURBINE TRIP

Under abnormal conditions occurring on either generator, grid or the reactor side of the plant such as load throw-off etc. the turbine tripping becomes necessary. Normally, it is performed by allowing 5% of the total steam through the main steam valve enough to keep the rotor rotating, and the rest 95% being by-passed to the condenser through by-pass valve (ref. Fig. 3.1).

In the simulation, this is performed in the following manner. The main throttle valve is closed by 95% of the initial value i.e.  $y$  is set equal to  $0.95 \times y_0$  to permit only 5% of the steam  $W_1$  in equation (3.91). In equation (3.62) representing the secondary valve, the area  $A_2$  is reduced to  $0.05 A_2$  so as to allow only 5% of the steam flow rate for the purpose of reheating in the reheater. The by-pass valve is modelled in such a manner that it allows the rest 95% of the steam to the steam condenser initially. This is represented by the following equation (4.2)

$$W_{BPV} = C_{BPV} A_{BPV} P \quad (4.2)$$

Where  $C_{BPV} \cdot A_{BPV}$  is evaluated by equating the initial 95% steam flow rate to the r.h.s. of equation (4.2) at the initial drum steam pressure. This by-pass valve is similar to the secondary steam by-pass valve. This assumption is valid only if down-stream pressure i.e.



condenser pressure is constant (an assumption already made) and the by-pass valve movement does not vary with time after the initial step. Now, in the simulation, eq. (3.99) is replaced by the following equation

$$W_S = W_1 + W_{PR} + W_{BPV} \quad (4.3)$$

As is to be expected steam flow from the drum,  $W_S$  drops down very fast, Fig. (4.4a) resulting in a steep increase of pressure Fig. (4.4 b). Since the steam flow rate  $W_S$  has decreased, the feed water flow  $W_{FW}$  also steeply decreases as per Eq. (3.34), Fig. (4.4 c). The increase of pressure results in voids to collapse, thus neutron power shoots up initially reaching 500 MW (Fig. 4.4 d) at 0.5 seconds. This causes the heat transfer rate  $Q$  and average fuel temperature  $T_F$  to increase as shown in Fig. 4.4 e and Fig.(4.4 f) respectively. The increase in  $T_F$  brings in negative Doppler reactivity feedback, thus bringing the neutron power back to lower values. Lastly, mechanical power Fig.(4.4 g) also decreases steeply since the flow rates through HP and LP turbine have decreased.

Some other tests which were simulated includes the following

1. 5 % increment in reactivity after 2 seconds,
2. 10 percent step increase in enthalpy,
3. pump speed stepped down to 25%,

4. pump trip and restart after 10 seconds,
5. main steam valve closure by 99.8 percent followed by two pump trip after 6 seconds,

they are not reported here as they showed expected behaviour and in order to present the condensed volume.

#### 4.3 CONCLUSIONS

A modified mathematical model for the BRP boiling water reactor nuclear power plant was studied. The model was developed on the basis of the physical laws such as mass, energy and momentum balance. This resulted in 21 non-linear differential equations and 75 algebraic empirical relationships. The model was simulated on the digital computer to validate it against the test results. The model showed good correspondence with the plant results. Some tests for which the plant results were not available, were also studied to observe the plant response. The emphasis was on studying large order transients on the model, however, some small order transients were also studied. This study showed that mathematical models could be developed to predict the plant behaviour for both small and large order disturbances.

#### 4.4 RECOMMENDATIONS FOR EXTENSION OF THE WORK

The work can be extended in two different ways:

##### A. SIMULATION STUDIES

- a. Further modifications can be introduced to replace the point kinetics by spatial kinetics to study the spatial effects.
- b. The voids can be suitably modelled to include 2nd order sweep effects.
- c. The grid generator sub-system can be coupled to the above plant to study the overall plant response.
- d. Variable time delays especially for pump trip tests etc. should be modelled.

##### B. MODERN CONTROL THEORY

- a. The plant model can be linearised and the feed back control approach can be applied to design the feedback controllers by using optimal modal control techniques.

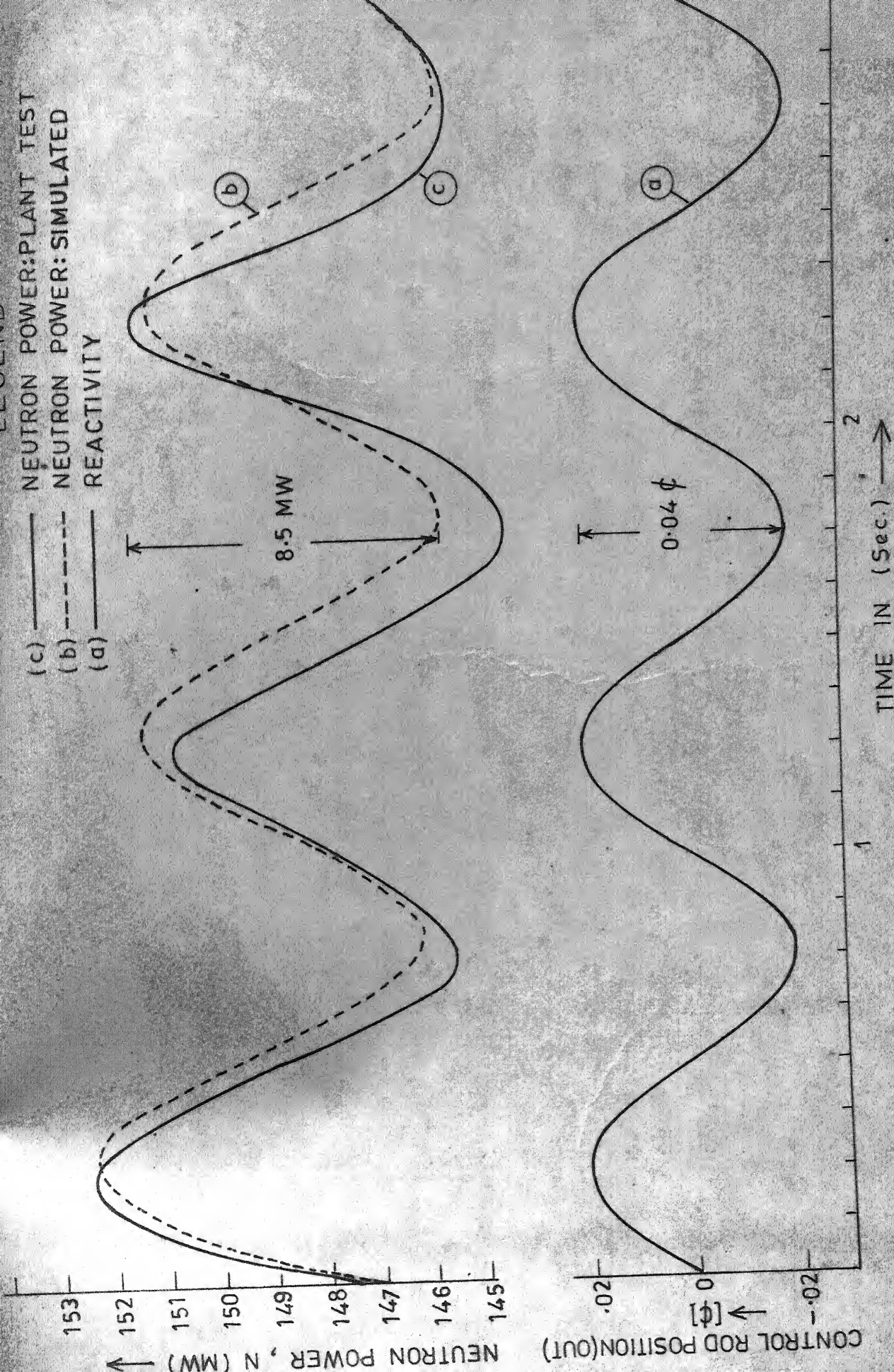


FIG. 4.1 CONTROL ROD OSCILLATION FOR 2  $\phi$  AT 1 CPS



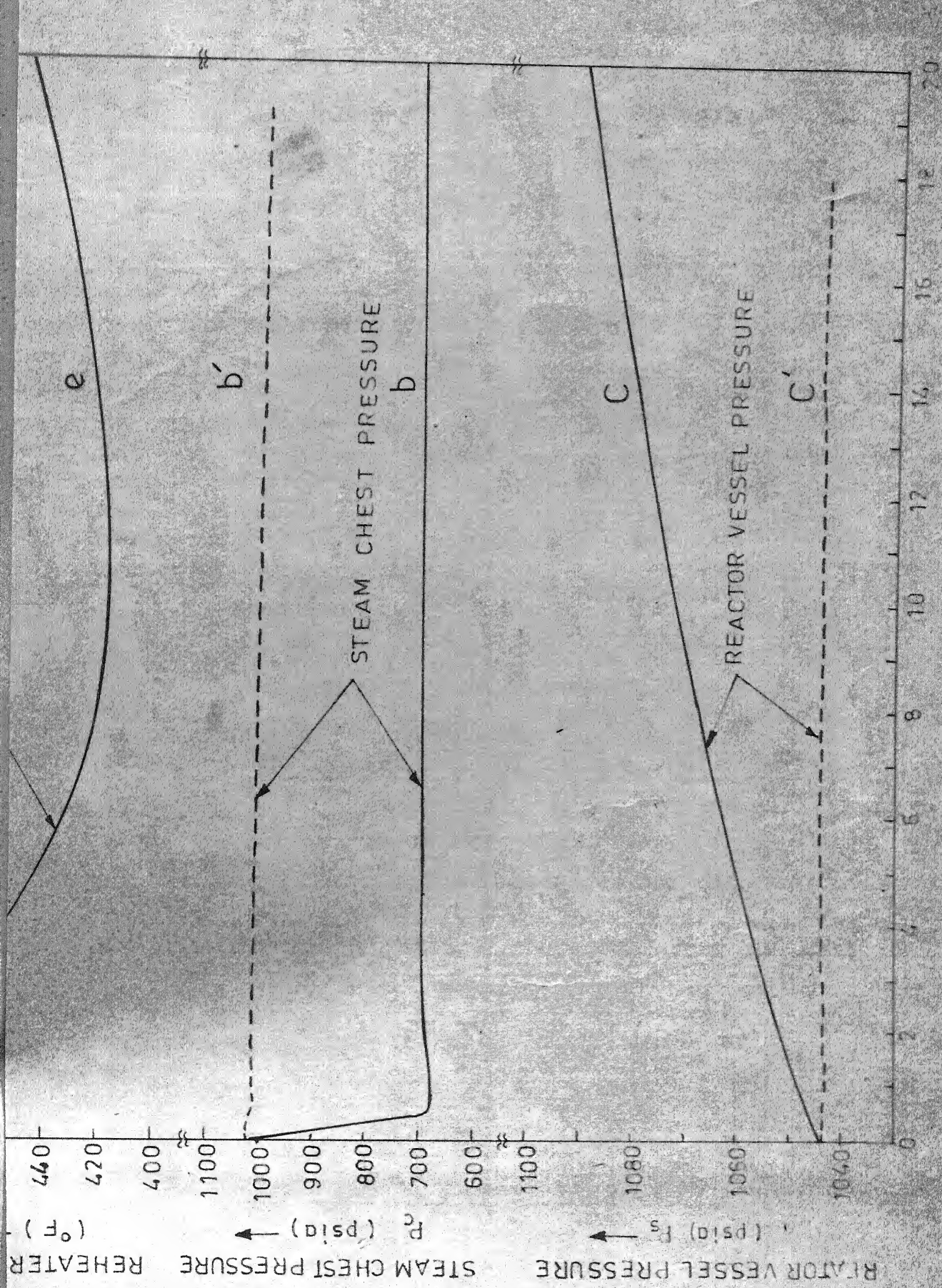


FIG. 4-2 20% & 80% STEP CHANGE IN THROTTLE VALVE

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